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ORIGINAL PAPER

Measurement Uncertainty Evaluation in Vickers Hardness Scale Using Law of Propagation of Uncertainty and Monte Carlo Simulation

I. Elizabeth[1]*, R. Kumar[1], N. Garg[2], M. Asif[2], R. M. Manikandan[1], Girish[1] and S. S. K. Titus[1]

[1]Force and Hardness Metrology Section, Physico-Mechanical Metrology Division, CSIR-NPL, New Delhi 110012, India

[2]Acoustics and Vibration Metrology Section, Physico-Mechanical Metrology Division, CSIR-NPL, New Delhi 110012, India

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Abstract: The Monte Carlo simulation (MCS) method for uncertainty evaluation of measurement results has gained popularity as an alternative to the method based on law of propagation of uncertainty (LPU) ever since the recommendation by JCGM through supplement JCGM 101:2008. In this paper, efforts have been made to compute the uncertainty in measurement of hardness in the Vickers hardness scale using MCS and LPU method. Three different hardness blocks of different hardness scales, namely HV1, HV10 and HV30, have been calibrated following the standard procedure in the primary Vickers hardness machine established in CSIR-National Physical Laboratory. The mean hardness values and the associated measurement uncertainty of the hardness blocks are computed using LPU and MCS methods. A comparison of the results obtained through LPU and MCS methods has been carried out. It is observed that there is a good agreement between findings from both the methods adopted, hence confirming that MCS method can be employed in the various fields of hardness metrology for evaluation of measurement uncertainty.

Keywords: Vickers hardness; Monte Carlo; Uncertainty; Metrology

1. Introduction

In metrology, the evaluation of uncertainty associated with any measurement result is very critical and the measurement result is complete only when it is accompanied by a measurement uncertainty. The measurement uncertainty parameter gives the statistical dispersion of the values attributed to the measured quantity. It accounts for the random and systematic errors which occur during measurement. Evaluation of measurement uncertainty is essential in order to ensure the metrological traceability of the measurement result and guarantee that the measurements taken are accurate and reliable. Also, measurement uncertainty evaluation becomes vital when decision has to be made based on measurement results and for quality assurance of products. Uncertainty evaluation of measurement plays an important role in production industries for maintaining improved tolerances and reducing the waste of production which occurs from machines whose accuracy is lacking to the tolerances of the process [1]. It also affects the international trade market where the quality of the products needs to be monitored [2].

According to the GUM document released by BIPM (*Bureau International des Poids et Mesures*) to harmonize the measurement uncertainty evaluation process across the world, the evaluation of measurement uncertainty is based on the law of propagation of uncertainties (LPUs) in which overall uncertainty of measurement result is obtained by taking into account the relative uncertainty component of each of the input quantities [3]. This approach has been applied for different measurement systems and is presently the most accepted methodology for uncertainty evaluation in metrology worldwide.

However, the LPU method of measurement uncertainty evaluation presented in GUM document is based on certain approximations and hence possesses certain limitations [3–5]. These approximations include the linearization of

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*Corresponding author, E-mail: induelizabeth@nplindia.org

the measurement model made by the truncation of the Taylor series, approximating the distribution of measurand to be a t-distribution even when the distributions can be of different types [6, 7]. Also, calculation of the effective degrees of freedom for determining the expanded uncertainty based on the Welch–Satterthwaite formula is still in debatable state [8]. The MCS method for uncertainty evaluation as per the JCGM 101:2008 guide (evaluation of measurement data—Supplement 1 to the "Guide to the expression of uncertainty in measurement"—propagation of distributions using a Monte Carlo method) [9] which is based on the propagation of probability distributions helps to overcome these limitations of LPU [3, 6]. The MCS method of measurement uncertainty evaluation basically involves the convolution of the probability distributions for the input quantities through the measurement model to generate a distribution for the output [10, 11].

In the present study, we have made a comparison between the evaluation of measurement uncertainty based on LPU and MCS methods for hardness measurement in Vickers hardness scale. For industrial applications, accurate measurement of hardness of materials plays an important role in materials testing, quality assurance and acceptance of materials for manufacturing. It is a prerequisite in many industries that the material should posses the desired hardness before being used in production. Hardness measurement is done mainly using three different scales, namely Rockwell, Vickers and Brinell hardness [12]. All these scales of hardness measurement involve making an indentation on the surface of the material being tested with an indentor of specific shape and dimensions (as per the corresponding standards) for a specific duration of time. The indentation is then measured, and the hardness value is assigned as per the standards. This study focuses on the hardness measurement using the Vickers hardness scale and the related uncertainty evaluation using two different methods—law of propagation of uncertainty (LPU) and the Monte Carlo simulation (MCS).

2. Vickers Hardness Measurement

The Vickers hardness is evaluated by making an indentation on the material to be tested with a specific force using a diamond pyramid-shaped indentor with specific dimensions as given in the standards for a specified amount of time. The Vickers hardness (HV) number is then determined by the ratio *F*/*A*, where *F* is the test force applied by the diamond indentor in kgf and *A* is the surface area of the resulting indentation in square millimetres. *A* can be determined by the formula [12, 13].

$$\text{Surface Area of Indentation } (A) = \frac{d^2}{2\sin\alpha/2} \tag{1}$$

where d is the arithmetic mean, in millimetres, of the two diagonal lengths d_1 and d_2 of the rhombus shape indentation formed on the material [13] and α is the mean angle between the opposite faces at the vertex of the pyramidal indenter (nominally 136°).

According to the ISO-6507 [13] standards, the Vickers hardness number (HV) is given by

$$\text{Vickers hardness number (HV)} = \frac{\text{Test Force (kgf)}}{\text{Surface Area of intendation (mm}^2)} \tag{2}$$

$$= \frac{1}{g_n} \times \frac{\text{Test force (N)}}{\text{Surafce Area of Indentation (mm}^2)} \tag{3}$$

$$= \frac{1}{g_n} \times \frac{F}{\frac{d^2}{2\sin(\alpha/2)}} \tag{4}$$

$$= \frac{1}{g_n} \times \frac{2F\sin(\alpha/2)}{d^2} \tag{5}$$

$$= 0.1891 \times \frac{F}{d^2} \tag{6}$$

where g_n is the standard acceleration due to gravity, g_n = 9.80665 m/s^2 which is the conversion factor from kgf to N [5], and α is the angle between the opposite faces at the vertex of the pyramidal indenter taken as 136°.

Figure 1 shows the model for Vickers Hardness measurement as derived from Eq. (6). It displays the factors contributing to measurement uncertainty in the hardness value being measured.

3. Primary Vickers Machine Established in CSIR-NPL

CSIR-National Physical Laboratory, New Delhi, is designated as the National Metrology Institute (NMI) of India by the Act of Parliament in 1950 and is the custodian of National Standards of measurements. NPL provides metrological traceability of various measurement parameters to industries, research institutes, government organizations like ISRO, DRDO, etc., across the country and

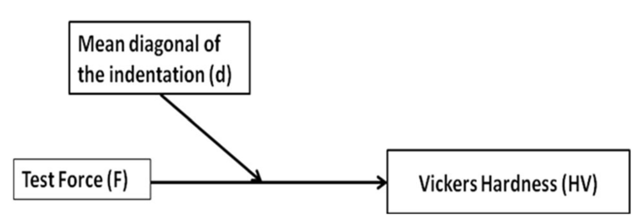

Fig. 1 Vickers hardness model showing the factors contributing to measurement uncertainty

hence helps in building the quality infrastructure of the country [14–17]. The Force and Hardness Metrology Section of CSIR-NPL maintains the primary standards of hardness—Vickers, Brinell and Rockwell. Different hardness scales are used for testing different materials as per their applications and need.

The Vickers hardness machine was established as per ISO 6507-3 standard for providing the national traceability in hardness measurement across the country [18]. The Vickers hardness machine used for measuring the hardness in the HV scale was designed and developed by LTF, Italy, and was installed at NPL, India. It consists of two systems, one for making the specific indentation on the surface of a test piece (Fig. 2) and another for measuring the geometrical dimension and determining the Vickers Hardness value of the test block (Fig. 3).

The part of the machine for making indentation comprises of calibrated precision dead weights for generating the test forces and a diamond indenter to make the indentation on a test surface of the hardness block. The required masses are selected on the loading frame from the loading basket to apply the desired test force, maintaining the appropriate testing cycle as per the international standard ISO 6507-3:2005 [13] using the machine. The loading basket can be moved vertically up and down along with the guide rail mounted vertically in an air-bearing shaft with the help of a pneumatic system which minimizes the shock, vibration/oscillation, if any, during the application of load. The required test load is thus applied on the hardness block

Fig. 2 Primary Vickers hardness machine at CSIR-NPL, New Delhi

Fig. 3 Indentation diagonal measuring system associated with the primary Vickers hardness machine at CSIR-NPL

by operating the loading basket as per the standard procedure. A suitable measurement system (Galvision) is used for measuring the indentation diagonal as depicted in Fig. 3. The indentation image is captured using a microscope having two eyepieces for viewing the object field which also comprises of a high-definition CCD for projecting the acquired image on a LCD monitor. On the plane containing the indentation to be measured, a Heidenhain linear encoder having the required resolution with 25 mm stroke is mounted coinciding with the indentation plane on the same axis. The diagonals of the indentation formed are measured using a software, and corresponding hardness values are assigned.

Direct verification of the machine has been carried out by evaluating the uncertainty of measurement in the generated force, the indenter geometry and the indentation measuring system. From these measurements, it is found that the machine exhibits a calibration and measurement capability (CMC) of $\pm$ 1.5% for HV1–HV3 scales and $\pm$ 1.0% for HV5–HV50 scales [18].

4. Measurement Uncertainty Evaluation of Vickers Hardness Using LPU

In the LPU approach, the measurement uncertainty is obtained by taking into consideration the relative uncertainty component of each input quantity in the model equation relating the input quantities to the output. If X_1,

X_2, …X_N are the input quantities and the output Y is a function of the input quantities, i.e. $Y = f(X_1, X_2, \ldots, X_N)$, then the uncertainty of Y depends on the relative uncertainties of the input quantities, the method adopted and procedure followed. Thus, the relative uncertainty components of different factors involved will collectively contribute to the overall uncertainty of Y [3, 19].

The general expression for uncertainty propagation is as follows [3, 6]:

$$u_Y^2 = \sum_{i=1}^{N} \left(\frac{\mathrm{d}f}{\mathrm{d}x_i}\right)^2 u_{x_i}^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{\mathrm{d}f}{\mathrm{d}x_i}\right)\left(\frac{\mathrm{d}f}{\mathrm{d}x_j}\right)\mathrm{cov}(x_i, x_j) \tag{7}$$

where u_Y is the combined standard uncertainty for the measurand Y and u_{xi} is the uncertainty for the ith input quantity. The second term in Eq. (7) is related to the correlation between the input quantities. If there is no correlation between them, equation is further simplified as:

$$u_Y^2 = \sum_{i=1}^{N} \left(\frac{\mathrm{d}f}{\mathrm{d}x_i}\right)^2 u_{x_i}^2 \tag{8}$$

For getting the expanded measurement uncertainty (U), the combined measurement uncertainty (u_Y) is multiplied by a coverage factor k, typically in the range 2–3, to obtain $U = k \times u_Y$. The coverage factor k is based on the level of confidence selected.

In the Vickers hardness measurement, the combined uncertainty associated with the hardness value takes into account the relative uncertainties of diagonal length (d), the test force values (F) and other parameters like uncertainties in the indenter geometry, the indentation measuring system, etc., which are accounted in the CMC of the machine [18].

5. MCS Method of Measurement Uncertainty Evaluation

MCS is a computerized simulation technique that relies on the statistical analysis and random sampling [20]. It employs the bootstrapping method, wherein it uses a random selection of combination of possible values of input quantities to propagate the distributions associated with these quantities [21]. This is one of the efficient techniques which is mostly used to solve strenuous physical and mathematical problems to get the solution with ease. This technique is generally used to estimate the measurement uncertainty by propagation of distribution [21, 22] and is thus a straightforward way to calculate the uncertainty of measurement in the field of metrology and provides approximation of the results provided by analytical method [3, 23]. The expanded uncertainty can be calculated by *MCS* with the confidence level of 95% and coverage factor (k) by the formula of $k = U(Y)/u_Y$, where $U(Y)$ and u_Y are expanded uncertainty and standard deviation of the simulated output values. When the skewness value of Y forecast distribution is near zero, the expanded uncertainty can be calculated as [24]:

$$U(Y) = \left(\frac{Y_{(1+p)M/2} - Y_{(1-p)M/2}}{2}\right) \tag{9}$$

where $Y_{(1+p)M/2}$ and $Y_{(1-p)M/2}$ correspond to the 2.5% and 97.5% percentiles of the sorted Y values for coverage probability (p). *MCS* has been extensively used in various fields of metrology such as in temperature [25, 26], pressure [27], length and dimension [10, 28] and force and hardness [6, 19] metrology to calculate the uncertainty.

Here, for the uncertainty evaluation of Vickers hardness scale, MCS method has been used. The steps followed for the uncertainty calculations are shown in the flow chart in Fig. 4. The model Eq. (6) for calculating the hardness value using Vickers hardness method is employed for calculating the measurement uncertainty for three different blocks, viz. HV1, HV10 and HV30. The MCS methodology was used to generate 2 × 10^5 random numbers in Microsoft Excel. The probability distribution function (PDF) was chosen to be normal for force and indentation depth parameters.

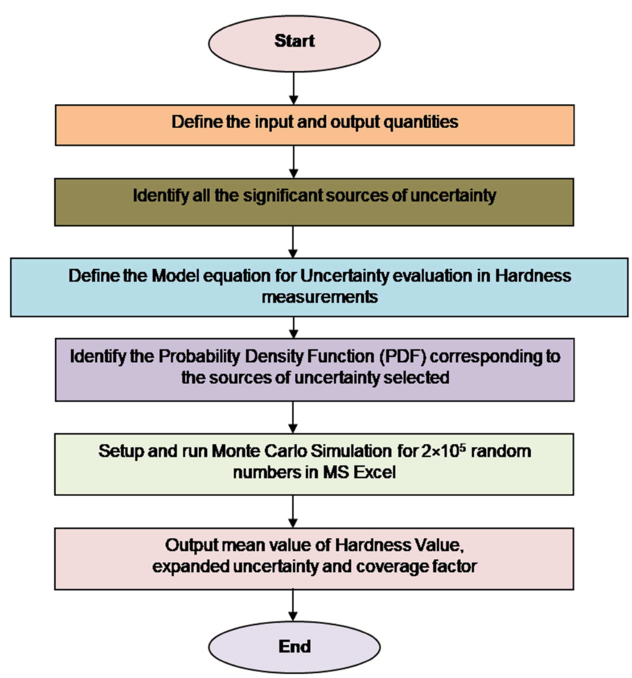

Fig. 4 Procedure for Monte Carlo simulation (MCS) for evaluating expanded uncertainty in hardness measurements using Vickers hardness machine

Three hardness blocks of different Vickers hardness scales HV1, HV10 and HV30 were investigated by MCS method. The results of the simulations are in detail discussed below. Further, a comparison of the results obtained through LPU and MCS is done from which we could find that both the results are in good agreement with each other.

6. Results and Discussion

In this section, we have discussed in detail about the measurement uncertainty evaluation using both LPU and MCS method of three hardness blocks on different Vickers scale, namely HV1, HV10 and HV30. All the blocks were calibrated following the ISO 6507-3:2005 standard procedure using the primary Vickers hardness machine at CSIR-NPL.

6.1. Case Study: HV1

A hardness block of Vickers scale HV1 has been investigated. The hardness block was calibrated in the primary hardness machine in the HV1 scale following the standard procedure. Since the Vickers hardness scale is HV1, the test force applied was 1 kgf. The indentation was made on the surface of the hardness block at five different places, and the diagonals of the indents formed were measured using the optical microscope. The corresponding readings obtained are given in Table 1. The primary Vickers hardness machine at NPL has a CMC of 1.5% at HV1 scale.

Using LPU method, the measurement uncertainty was calculated and the observations are as given below.

Nominal force applied	1 kgf or 9.807 N
Average length of the diagonal of the indentation	120.99 µm
Average hardness value	126.9
Standard deviation of the hardness values	1.50
Standard uncertainty of the hardness values (*t*-distribution)	1.53 (k = 2)
Standard uncertainty of the machine for HV1 (1.5%)	1.90
Expanded uncertainty of the hardness block by LPU	2.4

The measurement uncertainty was also calculated using MCS simulation. The results obtained from MCS method are given in Table 2, and a histogram showing the probability distribution function for the same is shown in Fig. 5.

Table 1 Hardness measurement on HV1 scale using primary Vickers hardness machine

Sl. no	Test force (F)	d_1 (µm)	d_2 (µm)	d (µm)	Hardness values
1	9.807	119.40	123.00	121.20	126.3
2	9.807	121.10	119.40	120.25	128.3
3	9.807	121.60	122.50	122.05	124.5
4	9.807	121.50	119.40	120.45	127.9
5	9.807	121.10	120.30	120.70	127.3

The low and high end points for the HV1 hardness block are marked by two vertical lines covering 95% interval.

6.2. Case Study: HV10

A hardness block in HV10 scale was investigated in this case study. The CMC of the HV10 scale is 1.0%. The block was calibrated on the primary hardness scale following the standard procedures. The observations obtained are summarized in Table 3, and the summary of the measurement uncertainty evaluation by LPU is also given. Summary of uncertainty evaluation by MCS method (Table 4) and the resulting probability distribution is given in Fig. 6.

Nominal force applied	10 kgf or 98.07 N
Average length of the diagonal of the indentation	165.41 µm
Average hardness value	678.0
Standard deviation of the hardness values	8.38
Standard uncertainty of the hardness values (t-distribution)	8.55 (k = 2)
Standard uncertainty of the machine for HV10 (1%)	6.78
Expanded uncertainty of the hardness block by LPU	10.9

Similar case study has been conducted for HV30 scale also. The details of LPU and MCS method of uncertainty evaluation are given in the Supplementary Document. The

Table 2 Summary of parameters obtained from MCS method for HV1 hardness block

Parameter	HV1
Mean value	126.8
Expanded uncertainty	2.4
Coverage factor	1.96
Low end point (in HV)	125.3
High end point (in HV)	128.3

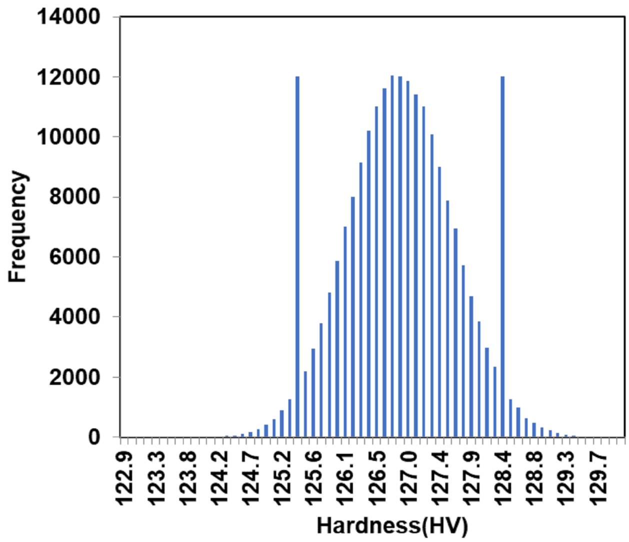

Fig. 5 Probability distribution function of Vickers hardness (HV1) scale using MCS method

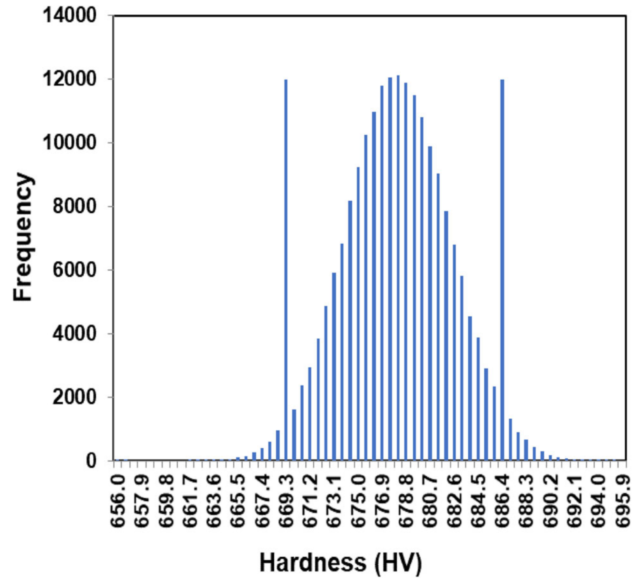

Fig. 6 Probability distribution function of Vickers hardness (HV10) scale using MCS method

Table 3 Hardness measurement on HV10 scale using primary Vickers hardness machine

Sl. no	Test force (F)	d_1 (µm)	d_2 (µm)	d (µm)	Hardness values
1	98.07	164.9	165.5	165.20	679.7
2	98.07	163.2	164.3	163.75	691.8
3	98.07	166.0	165.5	165.75	675.2
4	98.07	166.6	165.5	166.05	672.8
5	98.07	166.6	166.0	166.30	670.7

Table 4 Summary of parameters obtained from MCS method for HV10 hardness block

Parameter	HV10
Mean value	677.8
Expanded uncertainty	10.6
Coverage factor	1.96
Low end point (in HV)	669.7
High end point (in HV)	686.0

Table 5 Comparison of GUM/LPU method and MCS

Approach	Mean value of HV	Expanded uncertainty (in HV)
HV1		
GUM/LPU	126.9	2.4
MCS	126.8	2.4
HV10		
GUM/LPU	678	10.9
MCS	677.8	10.6
HV30		
GUM/LPU	451.6	7.8
MCS	451.5	7.6

summary of the uncertainty by two methods is compared in Table 5.

It is evident from Table 5 that the uncertainty evaluated by both the approaches is in good agreement with each other. For HV1 block, the expanded uncertainty in both the cases is the same. But in the case of HV10 block and HV30 block, there is a small difference in the uncertainties calculated with a maximum difference of 0.3 in the case of HV10. Hence, MCS method for uncertainty evaluation verifies the results obtained through LPU method.

7. Conclusion

This paper discusses about the LPU/GUM and the MCS methodology for measurement uncertainty calculations for hardness blocks in the Vickers hardness scale. Three different hardness blocks of HV1, HV10 and HV30 scales were investigated in this study. The blocks were calibrated on the primary Vickers hardness machine at CSIR-NPL following the standard procedure. The uncertainties associated with the measurement results were further calculated using two different approaches—LPU and MCS. The measurement uncertainties obtained from both the methods were compared and were found to be in good agreement with each other. A maximum difference of 0.3 was observed in the case of HV10 scale between both the methods, the values being within the permissible limits.

This study not only verifies the results obtained from LPU method but also shows that MCS can be employed in the various fields of force, hardness and torque metrology for evaluation of measurement uncertainty.

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