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ORIGINAL PAPER

# Measurement Uncertainty Evaluation Using Monte Carlo Simulation for Newly Established Line Scale Calibration Facility at CSIR-NPLI

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**Abstract:** High-precision line scales are probably the most common physical standards for length measurements. They are used as reference standards, transfer standards, direct length measurement devices and ordinary measures for adjustments in length measuring machines etc. Hence, in the current scenario, a robust and reliable line scale calibration infrastructure with high precision and flexibility is of indispensable need. Keeping this in view, an improved calibration facility for line scales, ranging from 300 to 1000 mm, has been established at CSIR-NPL India by combining coordinate measuring machines, vision metrology and displacement measuring laser interferometer. The present article describes line scale (400 mm) calibration setup, measurement procedure and measurement uncertainty evaluation. Here measurement uncertainty evaluation is carried out by using two different approaches, law of propagation of uncertainties (LPU/GUM) and Monte Carlo simulation. The measured mean values and expanded uncertainties obtained by using the above two approaches are found to be in good agreement.

**Keywords:** Line scale; Calibration; CMM; Vision metrology; Laser interferometer; Measurement uncertainty

## 1. Introduction

Precise dimensional measurements play a significant role in quality control, resulting in fair trade across the globe [1]. Being the National Metrology Institute (NMI) of our country, CSIR-NPL India unceasingly aims to reconnoiter the opportunities to provide accurate, precise and traceable dimensional measurements to our customers through apex level calibration services. In addition to this, continuous upgradation of existing calibration facilities and establishment of new calibration facilities is also an integral part of CSIR-NPL's mandate. To fulfill these objectives, we established a new line scale (ranging from 300 to 1000 mm) calibration facility at CSIR-NPL India. It has been evidently observed that on finalization of a measurement process, there are always some doubts associated with measurement results due to a lot of assumptions in measurements such as patchy measurement procedure, unknown environmental conditions influences on measurement and incomplete measurand information [2]. In order to provide quantitative and qualitative assurance in the measurement process, measurement uncertainty evaluation for calibration of line scale using this arrangement has been done by using two approaches LPU and MCS.

The law of propagation of uncertainties (LPU) approach for uncertainty evaluation is described by guide to uncertainty for measurement (GUM). In LPU approach, it is assumed that first of all the systematic errors are redressed and standard uncertainty associated with measurement result is described as a parameter. In this approach, a model equation is formulated by using input quantities and uncertainty components associated with each of them. It is assumed that the uncertainty of measurement output will be evaluated depending upon input quantities [2, 3]. The order of steps followed in measurement uncertainty evaluation using LPU/GUM is as mentioned below.

- Establish a mathematical relationship between output and input quantities in the form of model function
- Compute the estimated value of input quantity
- For each input estimate, compute the standard uncertainty
- Covariance evaluation associated with input estimates

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- Calculate the measurement results from functional relationship of input quantities
- Compute the combined uncertainty, expanded uncertainty and report associated expanded uncertainty with appropriate coverage factor

If output Y is the function of X (for example $X_1$, $X_2$, $X_3 \ldots X_N$ are the input quantities), then, in the absence of any correlation between input quantities, the combined standard uncertainty of output is expressed as follows:

$$u_{y=}^2 \sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \right)^2 u_{x_i}^2 \quad (1)$$

However, the GUM/LPU approach has certain limitations, such as the precondition of measurand model being linear, calculation of effective degree of freedom and LPU's reliance on central limit theorem [4]. As a consequence of these obvious constraints, Monte Carlo simulation (MCS) has been proposed as an alternative approach for measurement uncertainty evaluation by the virtue of being based on propagation of distribution of input quantities, not requiring the calculation of effective degree of freedom and sensitivity coefficient and being capable of giving actual probability distribution and graphical representation of output/measurand [5]. In LPU approach, the uncertainties are propagated as per their distributions while in MCS, the sources of uncertainties input quantities are propagated [6]. In MCS approach, a large number of pseudo random numbers with specified probability distribution functions (representing large number of trials to provide stability to measurement uncertainty) are generated which result in random sampling of various parameters [7, 8]. The steps followed in MCS approach for measurement uncertainty evaluation are discussed below [9]:

- Define input and output quantities
- Formulate model equation for measurement uncertainty evaluation
- Estimate input quantities probability density function
- Run Monte Carlo simulation
- Compile and report results

Past research work demonstrates that MCS proves to be more accurate and realistic approach for measurement uncertainty evaluation for nonlinear models also [10].

## 2. Measurement Setup and Procedure

The measurement setup for the current line scale calibration facility is established by combining coordinate measuring machines (Mitutoyo LEGEX 9106), vision metrology system (consisting of quick vision probe, CCD camera with a pixel calibration glass grid, LED optical illumination system and a software with automated edge detection capability) and traceable linear displacement measuring laser interferometer (Renishaw XL 80, including He–Ne laser head with wavelength 632.8 nm, polarizing beam splitter and two cube corner retroreflectors). First, the tactile probe of CMM is replaced by quick vision probe (QVP) (Fig. 1). The CCD camera of quick vision probe is calibrated by using a standard traceable calibration grid (Fig. 2) with vision metrology software.

Line scale is placed on CMM's movable bed. QVP height is adjusted and focused to achieve a sharp image of measurement marks/line graduations of the scale (Fig. 3). The measurement marks/line graduations along full length of line scale are aligned with CCD cross wire by moving CMM bed covering full scale range.

Laser head, interferometer optical components and displacement display unit are placed accordingly as in Figs. 4 and 5. The polarizing beam splitter (PBS) and cube corner reflector (CC1) are placed on fixed part of CMM bed while another cube corner reflector (CC2) is placed with scale on movable part of CMM bed. The zero scale mark is focused on CCD camera, and laser display unit is set at zero. Now the CMM bed is moved in order to focus the CCD camera


**Fig. 1** Quick vision probe

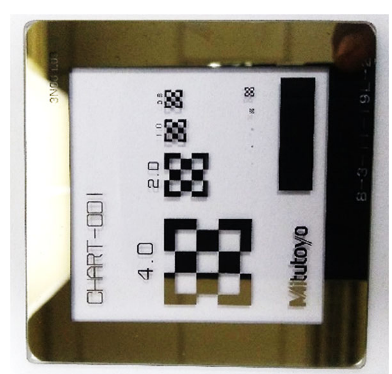


**Fig. 2** Pixel calibration grid

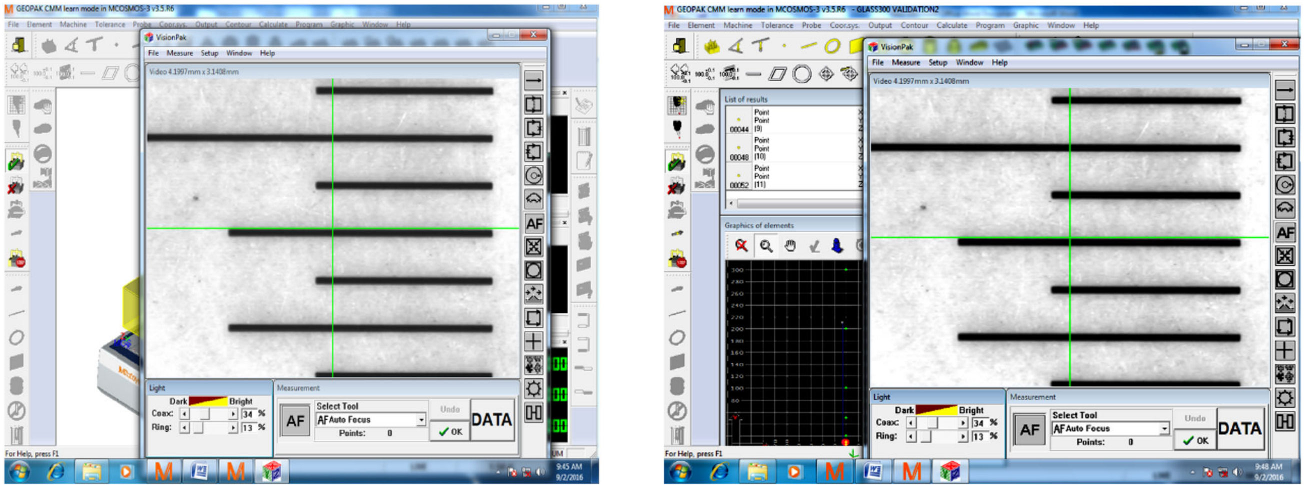

**Fig. 3** Line scale measurement marks on CCD camera

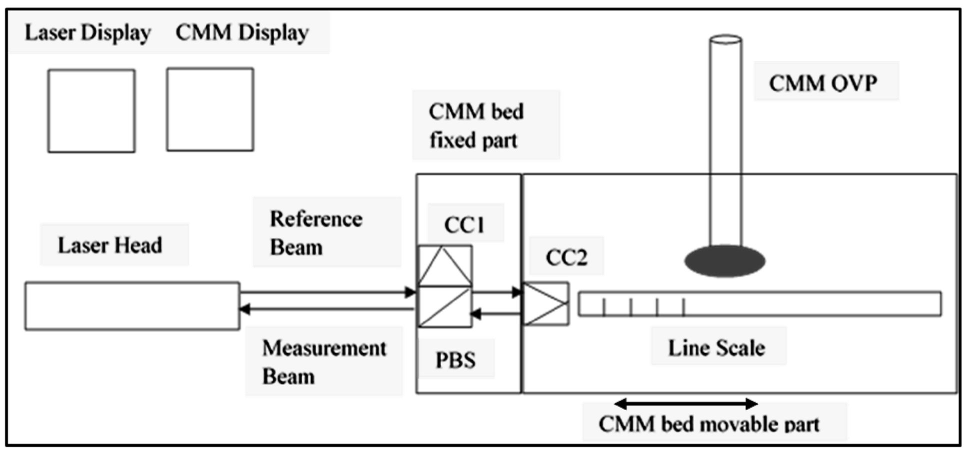


**Fig. 4** Schematic diagram of line scale calibration facility setup using CMM, vision metrology and laser interferometer

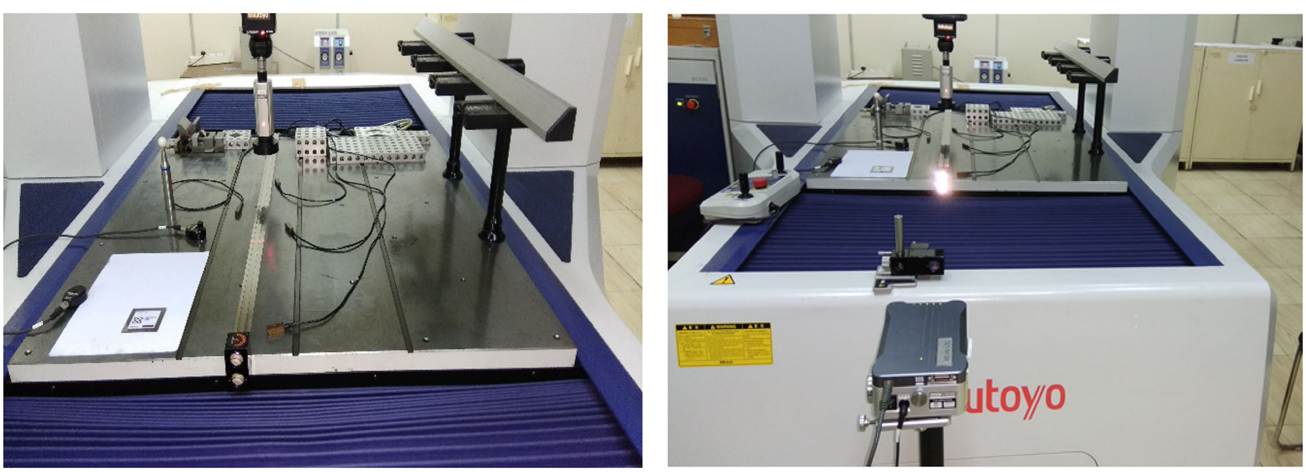

**Fig. 5** Actual photograph of line scale calibration facility setup using CMM, vision metrology and laser interferometer

on next measurement mark, and relative displacement of movable cube corner reflector (CC2) is observed form the display unit.

The relative displacement in this setup is measured by laser interferometer using phase shift interferometery is expressed by the following equation.

**Table 1** Observation table

| Scale nominal value (mm) | Laser reading (mm) | laser reading (mm) | laser reading (mm) | Laser reading (mm) | laser reading (mm) | laser reading (mm) | Average measured value (mm) | Error (mm) |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 10.0001 | 10.0006 | 9.9995 | 10.0000 | 9.9997 | 10.0003 | 10.0000 | 0.0000 |
| 20 | 19.9999 | 20.0007 | 19.9993 | 19.9999 | 19.9997 | 20.0002 | 20.0000 | − 0.0001 |
| 50 | 49.9996 | 50.0003 | 49.9995 | 49.9998 | 50.0000 | 50.0001 | 49.9999 | − 0.0001 |
| 100 | 99.9993 | 100.0002 | 99.9996 | 99.9999 | 100.0001 | 99.9997 | 99.9998 | − 0.0002 |
| 200 | 199.9992 | 200.0006 | 199.9997 | 200.0002 | 199.9998 | 199.9997 | 199.9999 | − 0.0001 |
| 250 | 249.9992 | 250.0007 | 249.9996 | 250.0002 | 249.9999 | 249.9998 | 249.9999 | − 0.0001 |
| 300 | 299.9992 | 300.0009 | 299.9996 | 300.0001 | 300.0000 | 299.9997 | 299.9999 | − 0.0001 |
| 350 | 349.9995 | 350.0009 | 349.9997 | 350.0002 | 350.0000 | 349.9997 | 350.0000 | 0.0000 |
| 400 | 399.9998 | 400.0017 | 400.0006 | 400.0006 | 400.0000 | 400.0000 | 400.0005 | 0.0005 |

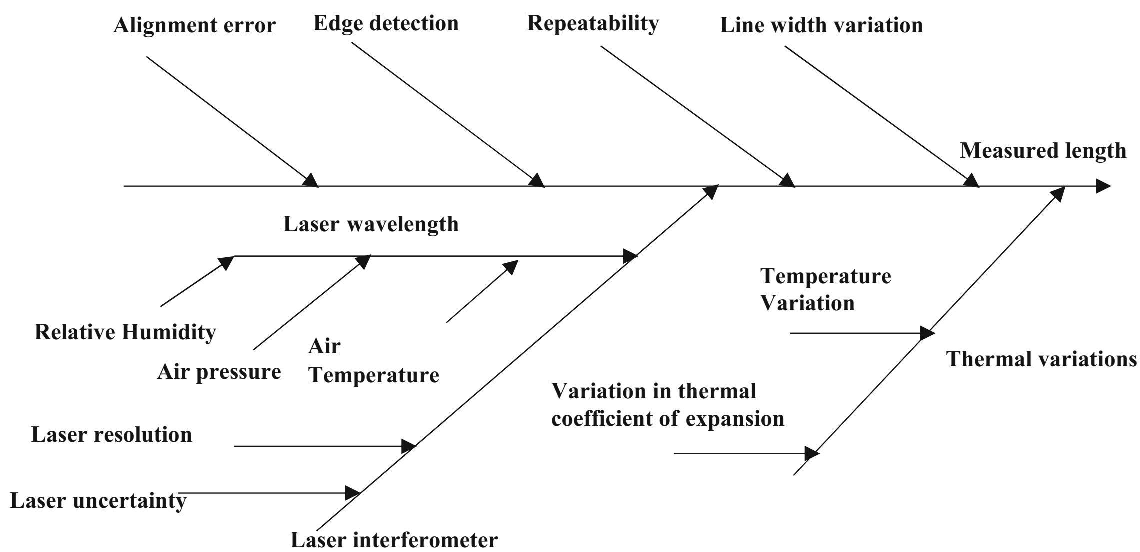


**Fig. 6** Fishbone diagram showing potential error sources

**Table 2** Uncertainty budget as per LPU

| Source $X_I$ | Units for limits | Limits ± | Type | Probability distribution | Standard uncertainty | Sensitivity coefficient | Uncertainty contribution $u_i$ ($y_i$) μm | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Length independent | Length dependent |
| VOL Compensation | ppm | | B | R | 0.20 | 1 | – | 0.2L |
| Material temperature variation | °C | 0.2 | B | R | 0.12 | 7.8 | – | 0.94 L |
| Variation in coefficient of thermal expansion | $°C^{-1}$ | 0.78 | B | R | 0.45 | − 0.3 | – | − 0.14L |
| Alignment error | μm | 0.2 | B | R | 0.12 | 1 | 0.12 | |
| Edge detection | μm | 1 | B | R | 0.58 | 1 | 0.58 | |
| Laser head uncertainty | μm | 0.05 | B | N | 0.03 | 1 | 0.03 | |
| laser resolution | μm | 0.1 | B | R | 0.06 | 1 | 0.06 | |
| Repeatability | μm | 0.17 | A | N | 0.17 | 1 | 0.17 | |
| Line width variation | μm | 0.4 | B | R | 0.23 | 1 | 0.23 | |
| Combined uncertainty | | | | | | | 0.66 | 0.97L |
| Expanded uncertainty (at k = 2) | | | | | | | 1.32 | 1.94L |

**Table 3** Monte Carlo simulation iterations for line scale calibration (for measurement point at 400 mm)

| Number of trials | Vol compensation (μm) | Material temp. variation (°C) | Alpha variation (°C$^{-1}$) | Alignment error (μm) | Edge detection (μm) | Laser head uncertainty (μm) | Laser head resolution (μm) | Line width variation (μm) | Relative linear displacement (mm) | Mean measured value (mm) | Expanded uncertainty (k = 2) (μm) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | − 0.03 | 19.84 | 7.65 | 0.04 | 0.86 | 0.03 | 0.08 | − 0.25 | 400.0003 | 399.9995 | 1.6 |
| 2 | − 0.10 | 20.16 | 8.27 | 0.04 | 0.64 | − 0.02 | 0.02 | − 0.09 | 400.0009 | 399.9994 | 1.6 |
| 3 | 0.11 | 19.82 | 8.48 | − 0.17 | − 0.36 | 0.05 | 0.05 | − 0.13 | 400.0008 | 399.9995 | 1.6 |
| 4 | 0.08 | 19.88 | 7.44 | 0.18 | − 0.89 | − 0.03 | 0.07 | 0.02 | 400.0001 | 399.9995 | 1.7 |
| 5 | − 0.17 | 19.85 | 7.54 | 0.07 | − 0.27 | 0.03 | 0.10 | − 0.40 | 399.9998 | 399.9995 | 1.6 |
| 6 | 0.01 | 20.12 | 7.40 | 0.15 | − 0.02 | 0.01 | 0.00 | − 0.01 | 400.0012 | 399.9994 | 1.7 |
| 7 | 0.15 | 20.19 | 7.49 | − 0.05 | 0.93 | − 0.04 | 0.10 | 0.06 | 400.0007 | 399.9995 | 1.6 |
| 8 | 0.04 | 20.05 | 8.06 | − 0.10 | 0.33 | − 0.05 | 0.07 | 0.08 | 400.0005 | 399.9998 | 1.6 |
| 9 | − 0.08 | 19.85 | 7.32 | 0.03 | − 0.26 | − 0.03 | 0.03 | − 0.24 | 400.0002 | 399.9998 | 1.6 |
| 10 | 0.02 | 20.08 | 7.59 | 0.14 | − 0.35 | 0.00 | 0.02 | − 0.39 | 400.0008 | 399.9999 | 1.6 |
| 11 | − 0.06 | 20.16 | 7.87 | 0.06 | 0.15 | − 0.01 | 0.03 | 0.16 | 400.0005 | 399.9995 | 1.7 |
| 12 | − 0.04 | 20.07 | 7.37 | 0.06 | 0.29 | 0.04 | 0.03 | 0.15 | 400.0012 | 399.9995 | 1.6 |
| 13 | 0.04 | 20.13 | 8.27 | − 0.08 | − 0.35 | 0.04 | 0.07 | 0.12 | 400.0017 | 399.9997 | 1.6 |
| 14 | − 0.18 | 19.90 | 7.62 | − 0.01 | − 0.89 | − 0.01 | 0.10 | − 0.02 | 400.0005 | 399.9995 | 1.7 |
| 15 | 0.07 | 20.17 | 7.20 | − 0.05 | 0.57 | − 0.05 | 0.01 | − 0.04 | 400.0000 | 399.9994 | 1.7 |
| 16 | − 0.18 | 19.83 | 8.06 | 0.12 | 0.42 | 0.02 | 0.04 | − 0.09 | 400.0005 | 399.9998 | 1.6 |
| 17 | − 0.12 | 19.94 | 7.46 | 0.02 | 0.59 | − 0.02 | 0.05 | 0.16 | 400.0010 | 399.9996 | 1.6 |
| 18 | − 0.03 | 19.96 | 7.11 | − 0.02 | 0.07 | − 0.02 | 0.06 | 0.31 | 400.0001 | 399.9993 | 1.6 |
| 19 | 0.04 | 20.16 | 8.30 | − 0.09 | 0.52 | 0.01 | 0.07 | 0.04 | 400.0011 | 399.9994 | 1.7 |
| 20 | 0.10 | 19.88 | 7.54 | 0.08 | 0.97 | − 0.05 | 0.00 | − 0.09 | 400.0004 | 399.9995 | 1.7 |

$$\Delta l = \frac{\Delta \emptyset \lambda}{4\pi n} \quad (2)$$

where ΔØ = phase difference; $\lambda$ = laser wavelength; and $n$ = refractive index

## 3. Results and Discussion

The newly established calibration facility can be used for calibration of line scales ranging from 300 to 1000 mm. The current article discusses calibration of a glass scale of length 400 mm using CMM, vision metrology and linear displacement measuring laser interferometer. Observation table for this calibration process done at room temperature 19.7°C and relative humidity 53% is given in Table 1.

Model function for measured value of various measurement points on line scale using this setup is formulated as:

$$\text{Measured Value} = \text{Nominal value} + \delta\text{vol} + \delta t + \delta\alpha + \delta a + \delta e + \delta\text{l w} + \delta\text{lu} + \delta\text{lr} \quad (3)$$

where δvol = correction due to wavelength of light compensation as per Edlen's equation; $\delta t$ = correction due to temperature variation; $\delta\alpha$ = correction due to variation in thermal coefficient of expansion; $\delta a$ = correction due to alignment error; $\delta e$ = correction due to edge detection error; δlw = correction due to line width variation; δlu = correction due to laser uncertainty; δlr = correction due to laser resolution

For measurement uncertainty evaluation, the potential error sources during measurement are identified, expressed in the form of Ishikawa fishbone diagram (Fig. 6), and a detailed uncertainty budget is prepared (Table 2) as per LPU approach.

Expanded uncertainty of measurement is = ± (1.32, 1.94$L$) µm, where $L$ is the nominal length of scale in meter.

Thus for 400 mm length (maximum range of scale), the expanded uncertainty is = ± 1.53 µm

In order to prove coherence between results of LPU and MCS, the measurement uncertainty evaluation for line scale calibration is also done by using MCS for the measurement point at 400 mm covering the full range of line scale. By generating random numbers for each error source 2, 00,000 iterations are done. Further standard deviation is calculated, and a probability distribution histogram at $k$ = 2 is obtained. A set of 20 iterations for all error sources is given in Table 3. The probability distribution histogram is shown in Fig. 7, and MCS result summary is given in Table 4.

A comparison of LPU result and MCS result is given in Table 5.

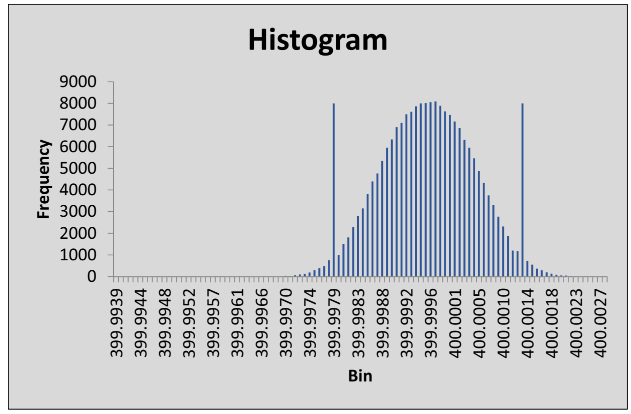


**Fig. 7** MCS histogram for line scale calibration (for measurement point at 400 mm)

**Table 4** MCS result summary for line scale calibration

| Parameters | MCS results for line scale calibration (for measurement point at 400 mm) |
|---|---|
| Mean measured value (mm) | 399.9996 |
| Expanded uncertainty (µm) at k = 2 | 1.64 |
| Low end point (mm) | 399.9979 |
| High end point (mm) | 400.0013 |

**Table 5** Comparison between LPU and MCS

| Approach | Mean measured value (for measurement point at 400 mm) | Expanded uncertainty (µm) at $k$ = 2 | En value |
|---|---|---|---|
| LPU/GUM | 400.0005 | 1.53 | 0.40 |
| MCS | 399.9996 | 1.64 | |

In order to compare and validate the two methods, En value is calculated and observed to be 0.40 satisfying the condition − 1 <= En <= +1.

## 4. Conclusion

In current manuscript, the new line scale calibration facility established at CSIR-NPL using CMM, vision metrology and linear displacement measuring laser interferometer is described distinctively. An investigation has been carried out for measurement error sources identification and measurement uncertainty evaluation during calibration of a line scale (length 400 mm) using this facility in accordance with two different approaches LPU/Gum and MCS in order to support JCGM recommendations. In pursuance of covering the maximum range of line scale, measurement uncertainty evaluations are done at 400 mm length. It has been observed that by using LPU the mean measured value is 400.0005 mm with expanded uncertainty ± 1.53 µm, and on using MCS, the mean measured value is 399.9996 mm with expanded uncertainty ± 1.64 µm. Results obtained from this analysis show satisfactory concordance between the two approaches.

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