

Experimental study on modulation of Stokes parameters on propagation of a Gaussian Schell model beam in free space

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Abstract: The effect on the Stokes parameters of a Gaussian Schell model beam on propagation in free space is studied experimentally and results are matched with the theory [X. H. Zhao, et al. *Opt. Express* **17**, 17888 (2009)] that in general the degree of polarization of a Gaussian Schell model beam doesn't change on propagation if the three spectral correlation widths δ_{xx} , δ_{yy} , δ_{xy} are equal and the beam width parameters $\sigma_x = \sigma_y$. It is experimentally shown that all the four Stokes parameters at the center of the beam decrease on propagation while the magnitudes of the normalized Stokes parameters and the spectral degree of polarization at the center of the beam remain constant for different propagation distances.

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References and links

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1. Introduction

In recent years, a lot of research has shown that the spectral degree of coherence and spectral degree of polarization may change on propagation of an electromagnetic Gaussian Schell model beam in free space [1–4], and also in different media [5–7]. The recently developed Wolf's unified theory of coherence and polarization [8], provides an intimate relationship between coherence and polarization properties of an electromagnetic beam and can predict changes in the coherence and polarization properties of the beam as it propagates.

In this paper we have verified experimentally the theoretical prediction [1, 9–11], that in general the degree of polarization of a Gaussian Schell model beam doesn't change on propagation in free space if the spectral correlation lengths δ_{xx} , δ_{yy} , δ_{xy} are equal to each other and the beam width parameters $\sigma_x = \sigma_y$. Schell model sources are the sources whose spectral degree of coherence $\mu^{(0)}(\xi_1, \xi_2, \omega)$ depends on the location of the two points only through the difference $\xi_1 - \xi_2$, of their position vector ξ_1 and ξ_2 [12]. It is shown that the magnitudes of the four Stokes parameters at the center of the beam change with the distance of propagation but the magnitude of the normalized stokes parameters and the degree of polarization remain unchanged. The study might be helpful in the area of free space optical communication (FSO) technology in which various polarization shift keying (PolSK) modulation schemes are used [13].

2. Theory

To prove the prediction made we would like to have a glimpse of some important results. Consider an electromagnetic Gaussian Schell model beam generated by a source in the plane at $z = 0$, propagating along the z -axis into the half-space $z > 0$ as shown in Fig. 1. The spectral density for the x and y components of the beam in the plane at $z = 0$ are given by,

$$S_i^{(0)}(\xi, \omega) = |A_{0i}|^2 \exp\left(-\frac{|\xi|^2}{2\sigma_i^2}\right), \quad (1)$$

where $i = x, y$ denotes the two orthogonal polarizations, A_{0i} is the amplitude, ω is the frequency, ξ is the position vector in the source plane and the parameter σ_i is related to the full width at half maximum of the beam. Here for simplicity it is assumed that $\sigma_x = \sigma_y = \sigma$.

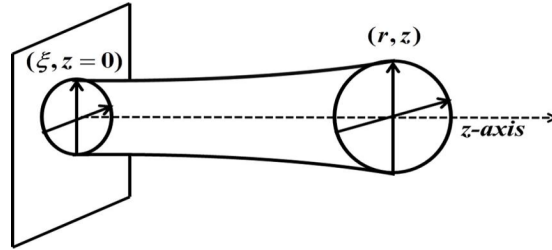


Fig. 1. Gaussian beam propagating in free space along the z -axis. The planes at $(\xi, z = 0)$ and $(\mathbf{r}, z)$ are source plane and observation plane respectively. Here ξ and $\mathbf{r}$ are two dimensional vectors.

For a Gaussian Schell Model beam, the spectral degree of coherence in the plane at $z = 0$, has the form [12],

$$\mu_{ij}^{(0)}(\xi_1, \xi_2, \omega) = B_{ij} \exp\left[-\frac{|\xi_1 - \xi_2|^2}{2\delta_{ij}^2}\right], \quad (2)$$

where $i, j = x, y$. The coefficients A_{0i} , B_{ij} and the parameter δ_{ij} are independent of position but may depend on frequency. In addition, these parameters also have to satisfy some constraints [12, 14, 15], i.e.

$$\text{for } \begin{cases} i = j, & B_{ij} = 1 \\ i \neq j, & |B_{ij}| \leq 1 \end{cases}, \quad (3a)$$

$$\text{and } \frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2} \ll \frac{2\pi^2}{\lambda^2}. \quad (3b)$$

The parameter B_{ij} is related to the degree of polarization [5, 12]. The cross spectral density matrix in the plane at $z = 0$ is given by,

$$W_{ij}^{(0)}(\xi_1, \xi_2; \omega) = \sqrt{S_i^{(0)}(\xi_1, \omega)} \sqrt{S_j^{(0)}(\xi_2, \omega)} \mu_{ij}^{(0)}(\xi_1, \xi_2, \omega). \quad (4)$$

From the beam propagation law [12], the cross spectral density matrix at any two points $\mathbf{r}_1$ and $\mathbf{r}_2$ in the plane at a distance ' z ' perpendicular to the direction of propagation is given by,

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2, z; \omega) = \frac{A_{0i} A_{0j} B_{ij}}{\Delta_{ij}^2(z)} \exp\left(-\frac{(|\mathbf{r}_1|^2 + |\mathbf{r}_2|^2)}{8\sigma^2 \Delta_{ij}^2(z)}\right) \exp\left(-\frac{|\mathbf{r}_2 - \mathbf{r}_1|^2}{8\delta_{ij}^2 \Delta_{ij}^2(z)}\right) \exp\left(-\frac{ik(|\mathbf{r}_2|^2 - |\mathbf{r}_1|^2)}{2\Phi_{ij}^2(z)}\right), \quad (5)$$

where $k = 2\pi/\lambda$ is the wave propagation constant and the quantities $\Delta_{ij}^2(z)$ and $\Phi_{ij}(z)$ are called beam expansion coefficients, given by,

$$\left. \begin{aligned} \Delta_{ij}^2(z) &= 1 + \frac{1}{(k\sigma)^2} \left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2} \right) \\ \Phi_{ij}(z) &= \left(1 + \frac{1}{\Delta_{ij}^2(z)} \right) z \end{aligned} \right\}. \quad (6)$$

The generalized Stokes parameters in terms of the elements of cross spectral density matrix in the observation plane $(\mathbf{r}, z)$ are given by [12],

$$\left. \begin{aligned} S_0(\mathbf{r}_1, \mathbf{r}_2, \omega) &= W_{xx}(\mathbf{r}_1, \mathbf{r}_2, \omega) + W_{yy}(\mathbf{r}_1, \mathbf{r}_2, \omega) \\ S_1(\mathbf{r}_1, \mathbf{r}_2, \omega) &= W_{xx}(\mathbf{r}_1, \mathbf{r}_2, \omega) - W_{yy}(\mathbf{r}_1, \mathbf{r}_2, \omega) \\ S_2(\mathbf{r}_1, \mathbf{r}_2, \omega) &= W_{xy}(\mathbf{r}_1, \mathbf{r}_2, \omega) + W_{yx}(\mathbf{r}_1, \mathbf{r}_2, \omega) \\ S_3(\mathbf{r}_1, \mathbf{r}_2, \omega) &= i[W_{xy}(\mathbf{r}_1, \mathbf{r}_2, \omega) - W_{yx}(\mathbf{r}_1, \mathbf{r}_2, \omega)] \end{aligned} \right\}. \quad (7)$$

The degree of polarization in terms of the Stokes parameters in the observation plane $(\mathbf{r}, z)$ is given by [12],

$$P(\mathbf{r}, z) = \sqrt{\left(\frac{S_1}{S_0}\right)^2 + \left(\frac{S_2}{S_0}\right)^2 + \left(\frac{S_3}{S_0}\right)^2}. \quad (8)$$

The Stokes parameters can be determined by putting $\mathbf{r}_1 = \mathbf{r}_2$ in the Eq. (7) and can be calculated from experimental measurements [16], using a combination of a quarter wave plate (QWP) and a linear polarizer in series with appropriate orientations of the fast axis and polarization axis respectively as shown in Fig. 2. Suppose $I(\theta, \phi)$ represents the intensity of

the beam recorded at CCD camera when the fast axis of the quarter wave plate makes an angle θ and polarization axis of the polarizer P_2 makes an angle ϕ with x-axis. The four Stokes parameters can be given by the following equations [16],

$$\left. \begin{aligned} S_0 &= I(0^\circ, 0^\circ) + I(0^\circ, 90^\circ) \\ S_1 &= I(0^\circ, 0^\circ) - I(0^\circ, 90^\circ) \\ S_2 &= I(0^\circ, 45^\circ) - I(0^\circ, 135^\circ) \\ S_3 &= I(45^\circ, 45^\circ) - I(45^\circ, 135^\circ) \end{aligned} \right\}. \quad (9)$$

3. Experimental details

We have used a single mode linearly polarized intensity stabilized 632.8 nm He-Ne laser (Mellus Griot) as shown in Fig. 2. Since the laser is vertically polarized, the polarization axis of the polarizer P_1 is oriented arbitrarily so that the experiment can be generalized for any orientation of the plane of polarization of the linearly polarized light. The ratio of peak intensities I_{0x} to I_{0y} in the plane at $z = 0$ is equal to 2.1 and the intensity profile of the beam in the plane is shown in the inset of Fig. 2. Black dots represent the experimental values while thick red curve represents its Gaussian fit. The full width at half maximum (FWHM) of the Gaussian fit (thick red curve) comes out to be 0.583 mm. The parameter σ used in Eq. (1) is calculated by $(FWHM \text{ of the Gaussian fit}) / \sqrt{8 \ln 2}$ which equals to 0.248 mm. Using Eq. (9), the Stokes parameters are measured at different points along the propagation direction.

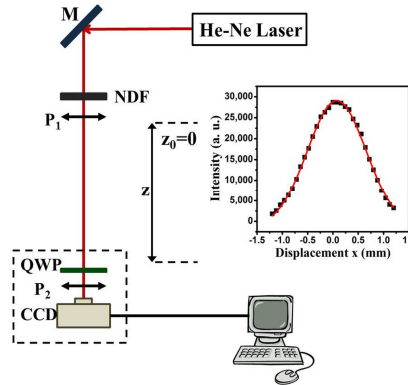


Fig. 2. Schematics of the experimental setup. M, NDF, QWP, CCD refers to the mirror, Neutral density filter, Quarter wave plate and CCD camera respectively. P_1 and P_2 are polarizers. The curve in the inset shows the intensity profile of cross section of the beam in the plane at $z = 0$. Black dots represent the experimental values while red curve represents its Gaussian fit. The FWHM of the Gaussian fit (red curve) is 0.583 mm.

The transverse coherence lengths δ_{xx} , δ_{yy} and δ_{xy} are measured using the HBT setup (Hanbury Brown and Twiss type setup, in which intensity-intensity correlations are measured rather than measuring the amplitude correlations [17]) by keeping the polarization axis of the polarizers P_2 and P_3 in the (x, x), (y, y) and (x, y) directions respectively as shown in Fig. 3. The beam is divided into two parts with the help of a 50-50 beam splitter. After that two polarizers P_2 and P_3 are placed in different arms of the HBT setup followed by two single mode fibers T_1 and T_2 connected to two avalanche photodiodes (APDs). One of the outputs of these APDs is fed to the 'Start' terminal and the other one is fed to the 'Stop' terminal (with a delay of 110 microsec) of a time to amplitude converter (TAC). The coincidence counts are measured by keeping one of the fiber tips fixed and other moving with the help of a computerized translational stage. The coincidence counts are recorded with a multichannel

analyzer (MCA) connected to the computer. The counts are taken for an integration time of 10s. The individual detector counts are kept low enough to eliminate accidental coincidence counts. For the laser beam the parameters $\delta_{xx} = \delta_{yy} = \delta_{xy} = \delta$. The curve showing the coincidence counts with the displacement between the two fiber tips T_1 and T_2 is shown in the inset in Fig. 3. Black dots represent the experimental values while thick red curve represents its Gaussian fit. The FWHM of the Gaussian fit (red curve) in the inset of Fig. 3, is 0.613mm. The parameter δ used in Eq. (2) is calculated by $(FWHM \text{ of the Gaussian fit})/\sqrt{8 \ln 2}$ which comes out to be 0.260mm.

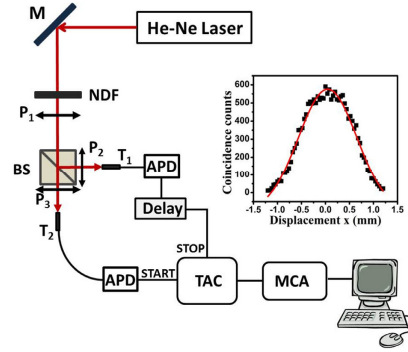


Fig. 3. Experimental setup for determining the transverse coherence length in the source plane at $z = 0$. BS is Beam splitter, P_1 , P_2 are polarizers, T_1 and T_2 are tips of two single mode fibers, APD refers to Avalanche photodiode, TAC is time to amplitude converter, MCA is multichannel analyzer. Curve in the inset shows the plot of coincidence counts with the displacement between the two fiber tips T_1 and T_2 . Square dots represent the experimental values while the red curve represents its Gaussian fit. The FWHM of the Gaussian fit (red curve) is 0.613 mm.

4. Results and discussion

The effect of propagation of the beam on Stokes parameter is shown in Fig. 4(a). The experimental values (black dots) fit reasonably well with the theoretically expected values (thick curves), within experimental uncertainty, with the parameters used in the experiment. After fitting theoretical curves the magnitude of B_{xy} comes out to ~ 0.91 (which is nearly equal to one since laser beam is linearly polarized) and $\arg(B_{xy})$ comes out to $\pi/2$ which is obvious because quarter wave plate introduces a phase difference of $\pi/2$ between x and y component of the electric field. Slight decrease in the magnitude of B_{xy} is due to experimental errors. The parameters B_{ij} , δ_{ij} and σ satisfy the constraints for a Gaussian Schell model beam given in Eqs. (3a) and (3b).

In Fig. 4(b), we see that the magnitudes of normalized Stokes parameters S_1/S_0 , S_2/S_0 and S_3/S_0 remain essentially constant and the experimental data agrees well with the theoretical prediction (thick curves) within experimental uncertainty. Since the spectral degree of polarization is square root of the squared sum of these normalized Stokes parameters as shown in Eq. (8), therefore it remains constant also. The constant behavior of the normalized Stokes parameters and the spectral degree of polarization on propagation of the beam is predicted [1, 12], because the transverse coherence lengths δ_{xx} , δ_{yy} and δ_{xy} are equal.

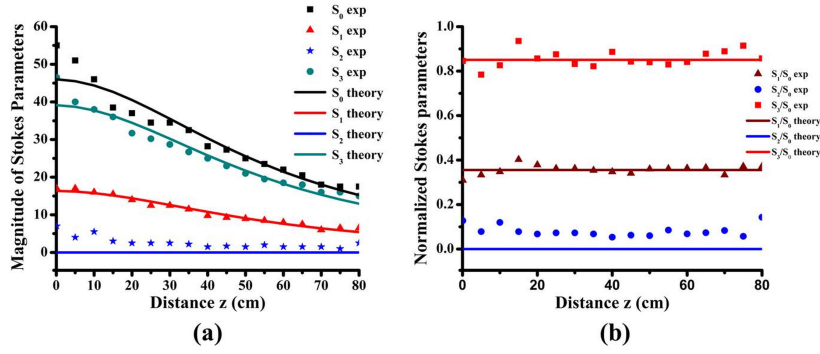


Fig. 4. (a) Variation in the magnitude of the Stokes parameters on propagation of the Gaussian Schell model beam in free space. Dots represent the experimentally calculated values while thick curves represent theoretical curves obtained from Eqs. (9) and (7) respectively. The experimental parameters $A_{0x}/A_{0y} = (I_{0x}/I_{0y})^{1/2} = 1.45$, $|B_{xy}| = 0.91$, $\arg(B_{xy}) = \pi/2$, $\sigma = 0.248\text{mm}$, $\omega = 3 \times 10^{15} \text{ sec}^{-1}$, $c = 3 \times 10^8 \text{ m/sec}$, $\delta_{xx} = \delta_{yy} = \delta_{xy} = \delta = 0.260\text{mm}$ are put in Eqs. (5) and (6) for getting the theoretically expected results. (b) Variation of normalized Stokes parameters S_1/S_0 , S_2/S_0 and S_3/S_0 on propagation of the beam in free space. Dots represent the experimentally calculated values while thick curves represent theoretical curves.

5. Conclusion

In summary, the modulation of the Stokes parameters at the center of a Gaussian Schell model beam on propagation is studied experimentally. It is shown that the magnitudes of the Stokes parameters decrease and the magnitudes of the normalized Stokes parameters and spectral degree of polarization remain unchanged on propagation of a Gaussian Schell model beam in free space. The decrease in the magnitude of the Stokes parameters is due to the diffraction effects. The experiment has been verified by putting the experimental parameters in the theory given above. It is shown that if the conditions that the three spectral correlation widths δ_{xx} , δ_{yy} , δ_{xy} are equal and the beam width parameters $\sigma_x = \sigma_y$, the normalized Stokes parameters and degree of polarization will not vary with propagation. However in the situation when $\delta_{xx} \neq \delta_{yy} \neq \delta_{xy}$, the normalized Stokes parameters and degree of polarization will vary with propagation of the beam in free space [12]. These results might have applications in free space optical (FSO) communication technologies where polarization shift keying (PolSK) modulation schemes are used.

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