A new method for estimating the critical current density of a superconductor from its hysteresis loop

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A B S T R A C T

The critical current density J_c of some of the superconducting samples, calculated on the basis of the Bean's model, shows negative curvature for low magnetic field with a downward bending near $H = 0$. To avoid this problem Kim's expression of the critical current density, $J_c = k/(H_0 + H)$, where J_c has positive curvature for all H, has been employed by connecting the positive constants k and H_0 with the features of the hysteresis loop of a superconductor. A relation between the full penetration field H_p and the magnetic field H_{min}, at which the magnetization is minimum, is obtained from the Kim's theory. Taking the value of J_c at $H = H_p$ according to the actual loop width, as in the Bean's theory, and at $H = 0$ according to an enhanced loop width due to the local internal field, values of k and H_0 are obtained in terms of the magnetization values $M^+(-H_{min})$, $M^-(H_{min})$, $M^+(H_p)$ and $M^-(H_p)$. The resulting method of estimating J_c from the hysteresis loop turns out to be as simple as the Bean's method.

1. Introduction

The Bean's method [1] of calculating critical current density (J_c) from the hysteresis loop of a superconductor is in use for over four decades. However, very recently we have pointed out that this method leads to a qualitatively incorrect behavior of J_c for low magnetic field (H) values in case of some samples of superconductors like MgB_2 and $PbMo_6S_8$ [2]. One of the feature of such superconductors is that in them the lower critical field (H_{c1}) is practically zero on the scale of the irreversibility field [2]. In fact, in the mentioned cases the Bean's method leads to a negative curvature of the J_c versus H curve for low H, which in some cases causes the critical current density to bend downward while approaching $H = 0$ from the higher H side. This is visible, for example, in the J_c of the 8% ND sample of MgB_2 of Cheng et al. [3], in the 5% and 7% ND samples of MgB_2 of Vajpayee et al. [4], and in the sample numbers 6 and 7 of Niu and Hampshire [5]. Fig. 1 demonstrates the behavior of the J_c of these superconducting samples in a schematic way. Since the lower critical field H_{c1} of these superconductors is practically zero on the scale of the irreversibility field [2], we expect that as soon as the magnetic field is applied to the superconductor, the vortices will start to move, thereby causing J_c to decrease with H [6]. This, is, however, true only for the cases when the vortex dynamics is not complicated, for example from effects like the fishtail effect [7] or like the discontinuous pinning centers [8]. It may, however, be noted that the fishtail effect will let J_c to increase with H only away from $H = 0$. This is clear, for example, from Fig. 2 of Jirsa et al. [7]. So, the fishtail effect is not obviously applicable in the present case of the MgB_2 and $PbMo_6S_8$ superconductors. As far as the case of discontinuous pinning centers with an over abundance of pinning centers [8] is concerned, J_c may increase with H in some cases even near $H = 0$. This is clear, for example, from Fig. 4 of Weinstein et al. [8]. Thus the increase of J_c with H mentioned above for the MgB_2 and $PbMo_6S_8$ superconductors near $H = 0$ may be due to discontinuous pinning centers with an over abundance of pinning centers. However, no such possibility is expressed by anyone of the groups of Refs. [3–5] in the MgB_2 or $PbMo_6S_8$ superconductors. We thus attempt to work out another source for the increasing of J_c with H near $H = 0$.

The main reason for the inadequacy of the Bean's method lies in the fact that in this method the effect of the local internal field H_i is neglected [9]. On the other hand, there is an essential involvement of the effect of the local internal field in the experimentally observed hysteresis loop. This means that the Bean's method should be modified so that the effect of the local internal field is well involved in the estimation of J_c. In Ref. [2] we have suggested a method for modifying the Bean's method so that the critical current density obtained from the hysteresis loop does not suffer from the negative curvature for low magnetic field, and at the same time it (the critical current density) gets inclined towards the Kim's critical current density up to the maximum possible extent. However, the method developed in Ref. [2] is somewhat difficult to use for the practical calculations of the critical current density, and at the same time it lacks its physical origin.

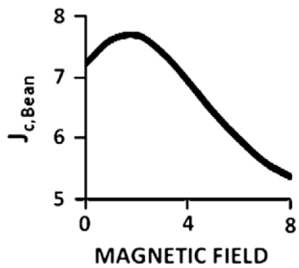

Fig. 1. Schematic representation of the negative curvature and downward bending of the critical current density J_c of a superconductor for H near zero. The units of J_c and H are arbitrary.

In this article we have made an effort by which one can estimate the critical current density from the hysteresis loop of a superconductor under the influence of the local internal field. The main points of our method are as follows. First of all we consider the critical current density to be given by the Kim's model [9], i.e. by

$$J_c = k/(H_0 + H) \tag{1}$$

Here k and H_0 are positive constants. We consider Eq. (1) because it guarantees, due to $\frac{d^2 J_c}{dH^2} = 2k/(H_0 + H)^3 > 0$, a positive curvature of J_c for all H. In comparison of the lengthy procedure of extending (contracting) the first-quadrant (fourth-quadrant) part of the hysteresis loop, as suggested in Ref. [2], it is much easier to use Eq. (1) from practical viewpoint.

Next we find out the constants k and H_0 from the hysteresis loop by specifying J_c for two values of the magnetic field. The first value of J_c corresponds to the full penetration field, and we take it according to the Bean's method, i.e. in terms of the actual width of the hysteresis loop at the considered field. The reason for this step is given in the next section. The second value of J_c corresponds to $H = 0$, and we have taken it in terms of the difference of the maximum and minimum values of the magnetization in the hysteresis loop. This means that the value of J_c, obtained in this way, will be maximum possible in terms of a given hysteresis loop. Although this way of specifying J_c contradicts the Bean's method, wherein one takes only the actual loop width, it is guided by the fact (see below) that the local internal field enhances the critical current density for $H = 0$. If, however, the maximum and minimum values of the magnetization in a hysteresis loop lie at $H = 0$, we shall obtain the Bean's value at $H = 0$ also.

The above mentioned two values of J_c, one corresponding to the full penetration field and the other to the zero field, are used in the left-hand-side of Eq. (1). Then the resulting two equations in k and H_0 are solved to find the values of these constants. In this way the Kim's expression of J_c (Eq. (1)) gets connected with the hysteresis loop resulting in a qualitatively correct method for the estimation of J_c from a given hysteresis loop.

The details of the above method are described in Section 2. In Section 3 we present an illustrative calculation so that the main points of the present method get cleared. Conclusions are drawn in Section 4.

2. Theory

In the Bean's method the critical current density is given by

$$J_{c,\mathrm{Bean}}(H) = G[M^+(H) - M^-(H)] \tag{2}$$

Here G is a geometric factor. For a cylindrical sample it is $3/2a$ [9], where a is the radius of the cross section of the sample. Below we shall limit to the case of a cylindrical sample only. However, generalization of the present method for other sample geometries is a straightforward task. The quantities $M^+(H)$ and $M^-(H)$ of Eq. (2) are respectively the positive and negative parts of the magnetization for a given magnetic field H.

If $J_{c,\mathrm{Bean}}(H)$ is to increase with H for low H, then, according to Eq. (2), we should have

$$\frac{dM^+(H)}{dH} - \frac{dM^-(H)}{dH} > 0. \tag{3}$$

Usually $\frac{dM^-(H)}{dH}$ is a positive quantity for all H (c.f. e.g., the caption of Fig. 9 of Niu and Hampshire [5]), but Eq. (3) requires it to be negative near $H = 0$. In fact, Eq. (3) requires that $\frac{dM^-(H)}{dH}$ should not only be negative, but also that its magnitude be larger than that of $\frac{dM^+(H)}{dH}$. Since Cheng et al. [3] and Niu and Hampshire [5] have not given the forms of the hysteresis loops in their work, and since Vajpayee et al. [4] have given only a part of the hysteresis loop near higher H, we consider a representative hysteresis loop on the basis of the critical current density given by these authors. A typical form of $M^+(H)$ and $M^-(H)$ of such a hysteresis loop is shown in Fig. 2 near $H = 0$. In this figure $M^-(H)$ is decreasing with H between $H = 0$ and $H = H_{min}$. Above $H = H_{min}$ $M^-(H)$ increases with H monotonically. From the behavior of J_c of the 8% ND sample of MgB_2 [3], and of the sample numbers 6 and 7 of $PbMo_6O_8$ [5] we expect that $H_{min} \approx 0.5$ T. The typical form of $M^+(H)$ and $M^-(H)$ of Fig. 2 corresponds to a $J_{c,\mathrm{Bean}}$ of the form given in Refs. [3–5]. Here it may be noted that the hysteresis loops based on the Kim's theory cannot lead the critical current density in the Bean's limit to bend downward near $H = 0$ (see below). Rather, we get only a negative curvature near $H = 0$.

The magnetic field H_{min} is the feature of the hysteresis loop. So, we consider it from the viewpoint of its connection with the constants k and H_0 of Eq. (1). In order to proceed in this direction we need a quantitative form of $M^-(H)$. As we are interested in a $M^-(H)$, which varies sharply between $H = 0$ and $H = H_{min}$, Figs. 6a–e and Eqs. (65) and (66) of Chen and Goldfarb [9] makes it clear that the required form of $M^-(H)$ will correspond to $H_0 \ll H_p$, where H_p is the full penetration field. Keeping this in mind, we consider Eq. (55) of Chen and Goldfarb for the case of $H_0 \ll H_p$. Then, making the transformation $M \to -M$ and $H \to -H$ in Eq. (55) of these authors we get

$$M^-(h) = \frac{H_p}{15}\left[-15h + 20h^3 - 8h^5 - 8(1-h^2)^{\frac{5}{2}}\right] \quad (0 < h < 1) \tag{4}$$

where

$$h = H/H_p \tag{5}$$

is the reduced magnetic field.

It is not difficult to find that the value of H, for which $M^-(H)$ of Eq. (4) has minimum value, is

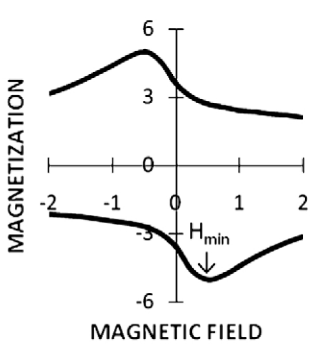

Fig. 2. A representative partial hysteresis loop for the 8% ND MgB_2 [3], 5% ND and 7% ND MgB_2 [4], and for the samples 6 and 7 of $PbMo_6S_8$ [5]. The units of M^+ and M^- are arbitrary, but H is in Tesla (T).

$$H_{min} = 0.2912H_p \quad (6)$$

This equation allows us to estimate H_p from the features of the hysteresis loop in the form of H_{min}. Here it may be noted that the value of H_p could have been easily obtained from the value of $M^-(H)$ for, say, $H = 0$, i.e. from $H_p = -15M^-(0)/8$ (c.f. Eq. (4)) by taking the value of $M^-(0)$ from the hysteresis loop. One can do so, but then the estimation of H_p is expected to be less reliable than that of Eq. (6). This is because the Kim's $M^+(H)$ and $M^-(H)$ never satisfy Eq. (3) near $H = 0$. We have found this result by using Eq. (51) of Chen and Goldfarb [9] for $M^+(H)$ in the limit $H_0 \ll H_p$. In fact what we find is that if the Kim's $M^+(H)$ and $M^-(H)$ are to satisfy Eq. (3), then H should be greater than 0.67 H_p. This certainly does not correspond to H near 0. So, we may say that the Kim's $M^+(H)$ and $M^-(H)$ will be relatively unreliable for providing $J_{c,\text{Bean}}$ similar to that of Fig. 1 as we approach $H = 0$. On the other hand, for the considered type of superconductors, H_{min} is expected to be considerably away from $H = 0$ so that the value of H_p is expected to be more reliable for this field. As we have mentioned above H_{min} is expected to be about 0.5 T for the 8% ND sample of MgB_2 [3], and for the sample numbers 6 and 7 of $PbMo_6O_8$ [5].

In order to take advantage of Eq. (6) from the viewpoint of estimating the value of the constants k and H_0 of Eq. (1) we use the fact that J_c of Eq. (1) and $J_{c,\text{Bean}}$ of Eq. (2) give the same set of values for $H \geqslant H_p$ (c.f. Fig. 6 of Chen and Goldfarb [9]). Then, considering in particular the critical density at the magnetic field H_p we may write

$$J_c(H_p) = J_{c,\text{Bean}}(H_p) \quad (7)$$

The left-hand-side of this equation includes k and H_0, while the right-hand-side involves information from the hysteresis loop in the form of H_{min} through Eq. (6). So, this equation provides a connection of the constants k and H_0 with the hysteresis loop. For a complete determination of k and H_0 we need one more relation of k and H_0 with the hysteresis loop. Since the hysteresis loop should give maximum critical current density for $H = 0$, we consider a relation of k and H_0 and the hysteresis loop through the critical current density for this value of the magnetic field. For this purpose, first of all we obtain the magnetization values $M^+(H = 0)$ and $M^-(H = 0)$ respectively from Eqs. (51) and (55) of Ref. [9]. The resulting expressions are given by

$$M^+(H=0) = -H_0 + \left(R^3 - H_0^3\right)Q - 2R^3/5ka \quad (8)$$

and

$$M^-(H=0) = -M^+(H=0). \quad (9)$$

Here

$$Q = 2\left(5ka + H_0^2\right)/15a^2k^2 \quad (10)$$

and

$$R = \left(2ka + H_0^2\right)^{1/2}. \quad (11)$$

From Eqs. (8) and (9) the loop width at $H = 0$ will be given by

$$\Delta M(H=0) = 2M^+(H=0). \quad (12)$$

Now, expanding R^3 in terms of powers of $2ka/H_0^2$ up to fifth order, and using Eq. (1) we find

$$J_c(H=0) = G\Delta M_{eff}(0) \quad (13)$$

where the effective loop width $\Delta M_{eff}(0)$ is given by

$$\Delta M_{eff}(0) = \Delta M(H=0)/(1 - \Delta H_i/4H_0). \quad (14)$$

Here $\Delta H_i = J_c a$ is the first-order difference between the local internal fields $H_i(0)$ and $H_i(a)$ [9].

When we compare Eq. (13) with Eq. (2) it turns out that $J_c(H=0)$ is increased over $J_{c,\text{Bean}}(H=0)$ by a factor of $(1 - \Delta H_i/4H_0)^{-1}$. It is clear that the origin of this enhancement lies in the role of the local internal field H_i. In fact, Eq. (14) reduces to Eq. (2) for $\Delta H_i = 0$. Although Eq. (14) indicates an effective enhancement of the loop width at $H = 0$, it is based upon the $2ka \ll H_0^2$ approximation, which is against the $H_0 \ll H_p$ requirement needed for Eq. (3). Thus, we use Eq. (14) only to learn that for a realistic value of the critical current density at $H = 0$, we must consider an enhanced loop width at $H = 0$. Guided by this finding, and noting that according to Eq. (1) $J_c(H=0)$ should be the maximum value of $J_c(H)$, we use the maximum possible value of the difference of $M^+(H)$ and $M^-(H)$ from a given hysteresis loop. According to Fig. 2 the maximum possible difference between $M^+(H)$ and $M^-(H)$ will be

$$\Delta M_{max} = M^+(-H_{min}) - M^-(H_{min}) \quad (15)$$

Thus, we specify the $H = 0$ critical current density by

$$J_c(H=0) = G\Delta M_{max}. \quad (16)$$

We are now in a position to find out the values of the constants k and H_0 of Eq. (1) from Eqs. (7) and (16). The result we obtain is

$$K = G\Delta M_{max}H_0 \quad (17)$$

and

$$H_0 = H_p\ \Delta M_p/(\Delta M_{max} - \Delta M_p). \quad (18)$$

From Eqs. (6), (17), and (18) it is clear that by obtaining $H_{min}, M^+(-H_{min}), M^-(H_{min}), M^+(H_p)$, and $M^-(H_p)$ from the hysteresis loops we can find the values of k and H_0. In this way we have got a consistent method for estimating the critical current density of a superconductor from its hysteresis loop. The important point of this method is that it is as simple to use as the Bean's method.

3. Results and discussion

In order to clarify the above method we have performed calculations by using Eqs. (1), (6), (7), (15), and (16) by considering the partial hysteresis loop of Fig. 2. We emphasize that this hysteresis loop is a representative of the hysteresis loops of Cheng et al. [3] and of Niu and Hampshire [5] near $H = 0$. In fact, as mentioned above, these authors have not given hysteresis loops in their articles. Although Vajpayee et al. have given hysteresis loop in Ref. [4], it is only for values of H much away from $H = 0$. Under such circumstances it is beneficial to use the hysteresis loop of Fig. 2 from the viewpoint of the illustration of the present method.

From Fig. 2 we find that $\Delta M_{max} = 10.0$ and $H_{min} = 0.5$ T. Then, using Eq. (6) we find $H_p = 1.72$ T. Corresponding to this value of H_p Fig. 2 gives $\Delta M_p = 5.65$. Notice that we are using arbitrary units

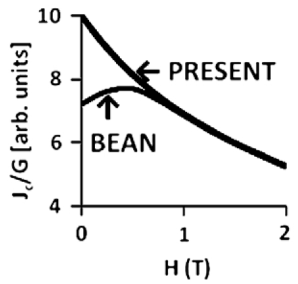

Fig. 3. Plot of critical current density divided by G corresponding to the partial hysteresis loop of Fig. 2 on the basis of the Bean's model (Eq. (2)) and the present model (Eqs. (1), (17), and (18)). The plot for Bean's critical density is marked with "BEAN", and that of the present method is marked with "PRESENT".

for ΔM_p, ΔM_{max} and J_c. From Eq. (18) we find $H_0 = 2.234$ T, and from Eq. (17) we get $k = 22.34$ G. Having obtained the values of k and H_0 we find that, corresponding to the hysteresis loop of Fig. 2, the critical current density will be given by

$$J_c/G = 22.34/(2.234 + H). \tag{19}$$

Using this equation values of J_c/G are plotted in Fig. 3. For comparison, values of the critical current density $J_{c,\text{Bean}}$ (Eq. (2)) divided by G are also presented in the same figure. We observe a drastic change as we approach $H = 0$. This means that the present method becomes increasingly important when we move towards $H = 0$. This will help us understand the vortex dynamics in a realistic way because the peak of the flux pinning density, which is proportional to the product HJ_c, is around $0.2H/H_{irr}$ in the considered MgB_2 and $PbMo_6S_8$ samples [3–5]. Here H_{irr} is the irreversibility field of the superconductor.

From the above illustrative calculation we see that $H_0 = 2.234$ T is larger than $H_p = 1.72$ T. This seems to be contradictory with the condition $H_0 \ll H_p$ used to obtain Eq. (4). Let us see what has actually happened. In the present method we have involved both the Kim's model (Eqs. (1) and (4)) and the Bean's model (Eq. (2)) through Eq. (7). The involvement of the Kim's model requires $H_0 \ll H_p$. On the other hand, the Bean's model requires $H_0 \to \infty$ and $H_p \to$ finite [9], i.e. $H_0 \gg H_p$. Thus the simultaneous involvement of the Kim's model and the Bean's model in the present method requires the two opposite conditions $H_0 \ll H_p$ and $H_0 \gg H_p$ to hold simultaneously. But, it cannot be possible. Under such circumstances a possible way is that none of these conditions are satisfied so that we have $H_0 \sim H_p$. This means that any one of these two parameters can be larger or smaller than the other one. It is in this sense that the relative values of H_0 and H_p found above on the basis of the hysteresis loop of Fig. 2 ($H_0 = 2.234$ T, $H_p = 1.72$ T) are justified.

4. Conclusions

In this paper we have presented a method for calculating the critical current density of a superconductor from its hysteresis loop. The dependence of the critical current density on the magnetic field is taken according to the Kim's model so that the curvature of the critical current density remains positive for all the magnetic fields. The two constants k and H_0, which appear in the expression of the Kim's critical current density, are connected with the features of the hysteresis loops by taking guidance from the effect of the local internal field. The resulting method provides not only a consistent variation of the critical current density with magnetic field, but also is as simple to apply as the Bean's method.

Although the present method provides a reasonably way for estimating the critical current density, and is qualitatively important for low H, there is a scope for an improvement of this method. This is because, as we have mentioned in Section 2, the magnetization given by the Kim's theory is not capable of describing the downward bending of the critical current density in the Bean's limit near $H = 0$. Therefore, there is a need for working out a more realistic theoretical method for the magnetization of a superconductor under magnetic field.

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