The Effect of Error in Position Co-ordinates of the Receiving Antenna on the Single-Satellite-Mode GPS Timing

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Abstract - **It is well established that the position co-ordinates of the receiving antenna should be determined precisely in advance for getting Time of the GPS receiver through a single GPS satellite technique. So it is desirable to know the extent of accuracy of the position co-ordinate required for a particular timing accuracy. In this paper, an analytical expression relating to the position error and the corresponding error in the Time of the GPS receiver has been derived. Time error of the GPS receiver caused by the error in position coordinates largely depends on the position of the satellite indicated by the respective elevation and azimuth of the satellite. To validate the derived formulation it is important to configure the experimental plan very judiciously. A special experiment has accordingly been conducted at National Physical Laboratory, New Delhi, India (NPLI). The observed data have been found to tally well with the derived relation.**

I. INTRODUCTION

GPS has evolved into a very powerful time transfer techniques with two important features. Time of the GPS receiver provides very high accuracy of few nanoseconds and Time of the GPS receiver has wide coverage as it is accessible anywhere in the world and anytime of the day. All clocks of GPS satellites are synchronized to the time scale of US Naval Observatory (USNO) is technically named as Universal Coordinated time of USNO in short UTC(USNO). Basic principle of GPS [1] is based on the pseudo ranges of the satellite which are measured at the receiving end by the travel time of the GPS signal. GPS Signal carries information including the orbital parameters of the corresponding satellites. From these parameters, the instantaneous position co-ordinates of the satellite are predicted. Basic purpose of GPS service is to provide the instantaneous values of three position co-ordinates of the antenna of the receiver and the time offset of the receiver clock. Determination of these four unknown parameters demands simultaneous measurements from four different GPS satellites. So for normal applications (i.e. both for position and time), one needs to receive signals from minimum four GPS satellites simultaneously. Mostly, GPS receivers work on this principle.

There exists another technique [2-4] of using GPS for the timing application. If three coordinates of the receiver position are known in advance then one may get time just by the measurement from a single satellite. This paper concentrates on this mode of GPS operation. This mode is mainly used by time keeping laboratories and the antenna has to be placed at a point whose coordinated are predetermined. The special timing receiver with the single satellite technique was developed right from the inception of the GPS. In course of time, the single satellite timing receiver has gone through many phases of development both in terms of hardware technology and software improvisations. This special type of timing receiver is currently used widely by timing keeping laboratories contributing to the generation of UTC coordinated by BIPM. Many time transfer techniques like common view mode , PPP mode , carrier phase technique etc are being evolved based on the special timing receiver utilizing the single satellite mode. Most of the timing laboratories are equipped with this type of receivers.

For getting Time of the GPS receiver through a single GPS satellite mode, it is necessary to know the exact position of the receiver's antenna. Position can be determined precisely with help of a geodetic receiver coupled with powerful software and utilizing IGS products at a later date. But timing laboratory may not be equipped with such receiver. So it would be quite interesting and appealing if some methodology could be worked out to determine precisely the position coordinates of the receiving antenna based on the timing receiver. In order to address this issue, finding an analytical expression for the effect of the position error on Time of the GPS receiver relating the corresponding satellite position would be quite useful. This is one objective of this paper. Second objective of this paper is to refine the position coordinates of the receiver's antenna with the help of this analytical expression coupled with the corresponding observed GPS data This paper attempts to derive the referred analytical expression A special experiment has also been conducted at National Physical Laboratory, New Delhi, India (NPLI) to corroborate the derived relation. The observed data have been analyzed and presented in this paper. An effort has been made to refine the position coordinates of the location utilizing the observed variations of time error with the relative position of the satellites. The paper also elaborates this methodology of refinement of co-ordinates.

II. ANALYTICAL EXPRESSION

The error in GPS timing operated in the single satellite mode is directly correlated to the range error arising out of the error in position coordinates of the receiving antenna. In order to derive an analytical expression of the correlation, let us assume that in earth centered fixed coordinate systems of WGS84, the exact coordinates of the receiving antenna are x^o, y^o and z^o. The coordinates have been determined in advance and are fed to the GPS receiver in advance are x, y, and z. Thus, $x-x^o$, $y-y^o$ and $z-z^o$ represents the error in the position coordinates. Because of the error in the coordinates, there will be range error leading to the timing error. Now let us assume that S is the position of GPS

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satellite in horizontal coordinate system with respect to the actual position O of the receiving antenna as the origin (Fig.1). X & Y axes lie north-ward and east-ward respectively and Z axes towards zenith. Let the P is the position of the antenna as determined. $\overline{OP}$ becomes the Position error vector ($\vec{E}$). OP is very small compared to OS. Therefore it is quite logical to assume that, OS and PS are parallel. OP' is projection of OP on satellite range vector OS. Thus OP' is range error (ΔR) (i.e. $\Delta R = OP' = OP\cos\Phi)$ caused by the position error.

$$\Delta R = \left|\vec{E}\right| \cos \Phi = \left|\vec{E}\right| \frac{\vec{S}.\vec{E}}{R} = u\vec{S}.\vec{E} \tag{1}$$

$u\vec{S}$ =Unit vector of the direction of satellite S from the origin O.

Satellite S, has Elevation (e) and Azimuth (a) and Range (R) with respect to the receiver position. In order to evaluate the ΔR, it is necessary to transform the vectors to horizontal coordinate system as shown in Fig. 1. The satellite position represented by e, a and R may be transformed [7] to the horizontal system as

$$\overline{S} = \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = R \begin{bmatrix} Cos\ (e).Cos\ (a) \\ Cos\ (e).Sin\ (a) \\ Sin\ (e) \end{bmatrix}$$

$$= R\vec{S} = Ru\vec{S} \tag{2}$$

Similarly the errors in position coordinates (x-x^o, y-y^o and z-z^o) may also be transformed [7] as

$$\vec{E} = \begin{bmatrix} -Cos(L)Sin(B) & -Sin(L)Sin(B) & Cos(B) \\ -Sin(L) & Cos(L) & 0 \\ Cos(L)Cos(B) & Sin(L)Cos(B) & Sin(B) \end{bmatrix} \times \begin{bmatrix} x - x^0 \\ y - y^0 \\ z - z^0 \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{3}$$

where L and B are the latitude and the longitude of the position of the antenna (O in Fig.1)

By combining equations (1), (2) and (3), the range error may be given by

$$\Delta R = E_x.Cos(e).Cos(a) + E_y.Cos(e).Sin(a) + E_z.Sin(e) \tag{4}$$

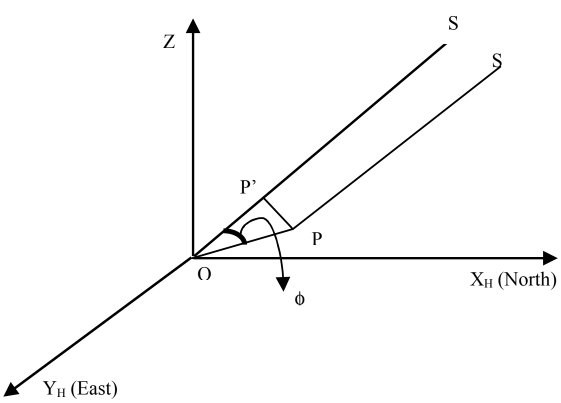

Fig.1. Position Error Vector in Horizontal Coordinate System

In single satellite GPS mode the corresponding time error (ΔT in nanoseconds) will be directly related as

$$\Delta T = \frac{\Delta R}{c} = \frac{1}{30}[E_x.Cos(e).Cos(a) + E_y.Cos(e).Sin(a) + E_z.Sin(e)] \tag{5}$$

Here, ΔR is in cm

From equation (3) it follows that

$$E_x = -Cos(L)Sin(B).\Delta x - Sin(L)Sin(B).\Delta y + Cos(B).\Delta z \tag{6}$$

$$E_y = -Sin(L).\Delta x + Cos(L).\Delta y \tag{7}$$

$$E_z = Cos(L)Cos(B).\Delta x + Sin(L)Cos(B).\Delta y + Sin(B).\Delta z \tag{8}$$

Where Δx=x-x^o, Δy=y-y^o, Δz=z-z^o

From equation (5) coupled with equations (6), (7), (8) the timing error ΔT may be determined.

III. EXPERIMENTAL PLAN

To validate the above formulation it is important to configure the experimental plan very carefully. If we measure the difference between the Time of the GPS receiver and the reference standard time, the difference becomes the error in Time of the GPS receiver (dT). It is important to note that this error [5] is contributed not only by the error in coordinate of the receiving antenna (e_{pos}) but also by the error of the satellite ephemeris (ρ_{eph}), ionosphere (d_{ion}) and troposphere(d_{trop}) model errors, multipath error (α_{mul}), receiver noise (r_{nos}) and the satellite clock error(dt_{sat}) as given by

$$c \cdot dT = e_{pos} + \rho_{ephe} + c \cdot dt_{sat} + d_{ion} + d_{trop} + \alpha_{mul} + r_{nos} \tag{9}$$

It is almost impossible to separately quantify the contribution to the time error by the respective factors. So the contribution of the position error to Time of the GPS receiver error cannot be found out from the measurement of one GPS receiver. To circumvent this, a special experiment as shown in Fig.2. has been planned. Two special GPS timing receivers (say, Receiver A and Receiver B) of the same make are used. Two antennas are placed side by side (separated by the distance of only 3.2 meters, in this case) and their co-ordinates are known precisely. Two receivers used here are special in the sense that they have built-in time interval counters with a resolution of 100 ps. They compare the Time of the GPS receiver with respect to 1pulse per

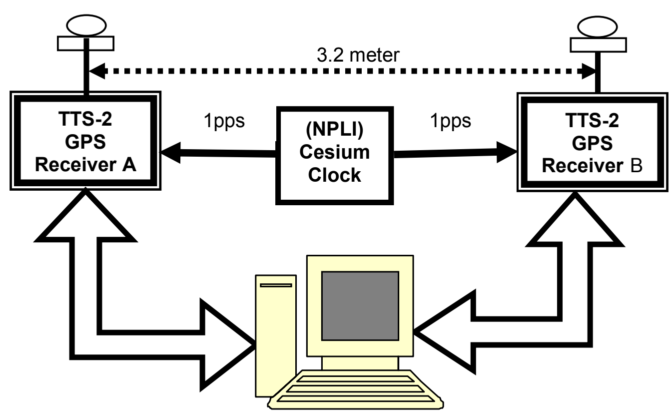

Fig.2 . Experimental Set-Up for the Special Experiment Campaign

second (1pps) externally fed from the reference clock. In this set up both receivers are fed with 1pps from the common source of the master Cesium clock (high performance) of NPLI. Each receiver records the time difference between Time of the GPS receiver and the externally fed 1pps (say for receiver A denoted as ($Ref\text{-}GPS_A$), the elevation (e), azimuth (a) and PRN number of the corresponding satellite and the time and date of the observation. The data from the two receivers are recorded in the computer for further analysis. Two receivers records data independently. For analysis, data from both the receivers are sorted for strict common view (i.e. for common satellite at the same time of observation). From the common view data, values of the ($Ref\text{-}GPS_A$) – ($Ref\text{-}GPS_B$) are calculated. They are denoted by δt. As two antennas are almost collocated, the errors in ionosphere and troposphere delay would be exactly same for Receiver A and Receiver B in strict common view. Two antennas are located at a place whose surroundings are quite clear eliminating the chance of multipath. So δt would be contributed only by the noise of the two receivers and by the error of the coordinates of the two antennas. So δt may be derived from equation (9) as

$$c\cdot\delta t = c\cdot(dT_A - dT_B) = e_{pos}(A) - e_{pos}(B) + r_{nos}(A) - r_{nos}(B) \qquad (10)$$

IV. ANALYSIS OF THE OBSERVED DATA

The position coordinates are fed to the respective receivers. δt, e, a and the corresponding time of observations are recorded for few days. The values of the δt have been found to have 1σ of 2.5 ns which, according to equation (10), may be primarily contributed by the noise of the receiver assuming the position coordinates have no error.

One way to validate the formulation given in equation (5), is to introduce deliberate error in position coordinates in one of the antennas substantially so that the contributions to the time error by the errors in the position coordinates is much higher than that (~2.5 ns) by the receiver noise. In order to satisfy this condition, one special observation was planned. The receiver B is fed with a new set of coordinates which have deliberately moderate errors. In such situation, δt will be primarily contributed by the position error of antenna B, the other factors in equation (10) being comparatively insignificantly small. Thus, in such situation, equation (10) reduces to

$$\delta t \approx \frac{e_{pos}(B)}{c} \qquad (11)$$

In these observations, position errors in receiver B, are:
Δx = 5099.47 meters, Δy = 1091.78 meters, Δz =4100.07 meters, For these values equation (5) reduces to

$$\delta t = \frac{1}{30}(23534.09335\cdot Cos(e).Cos(a) - 8637.679238\cdot Cos(e).Sin(a) - 4.765546786E+03\cdot Sin(e)) \qquad (12)$$

The variations of δt (based on equation (12)) with the azimuth for fixed values of the elevation have been plotted in Fig.3. It is impossible to generate similar graphs with the experimental data, as it is quite unlikely to get a good amount of data corresponding to any particular value of elevation angle. Rather, it is more pragmatic to look for a large number of data for elevation angle within a certain range of values. Keeping this in mind, the experimental data, for example, has been sorted for all

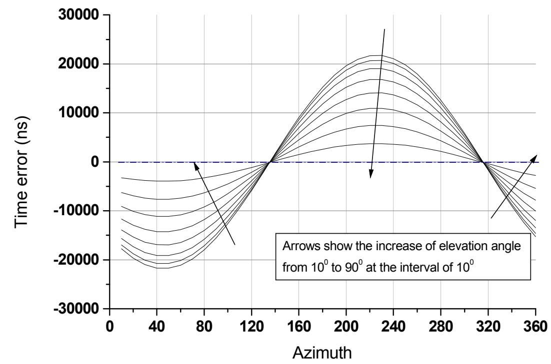

Fig.3. Variation of Time Error based on Analytical Derivation (theoretical) for Deliberate Position Error.

values of elevation in the range of 50° to 70° to validate the graphs of Fig.3. The values of δt, thus sorted, are plotted against the corresponding values of azimuth as shown in Fig.4. The solid line in Fig.4 is the corresponding theoretical values following equation (12) for the elevation angle of 50° and 70°. It is interesting to note that experimental data lies well inside the theoretical lines. Similar sorting of data have been made for corresponding data for elevation in the range of 10° to 30° and have been plotted in Fig.5. Critical inspections of the experimental data as discussed above exhibit excellent match with theoretical prediction. This leads one to confirm the validity of the relation shown in equation (5).

V. REFINEMENT OF POSITION CO-ORDINATES

The close inspection of the expression of equations (5-8) suggests that the time error data may be utilized to improve the correctness of the position coordinates of a place. To establish this proposition, one special experiment has been conducted to refine the coordinate of a location whose coordinates are not known correctly. The GPS timing receiver which has been placed on this location has been made to operate in the single satellite mode. To start with some nominal values of coordinates (i.e. x, y, z) have been fed into the receiver. The data has been recorded for few days in a format as shown in Table I.

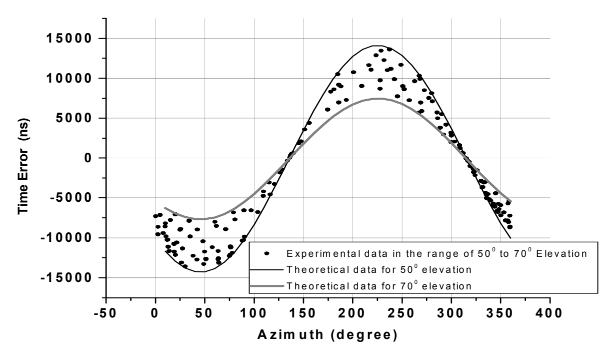

Fig.4. Experimental Observations of Time Error within the elevation range of satellite between 50^0 to 70^0 with theoretical values in solid line for elevation 50^0 & 70^0 for given position error.

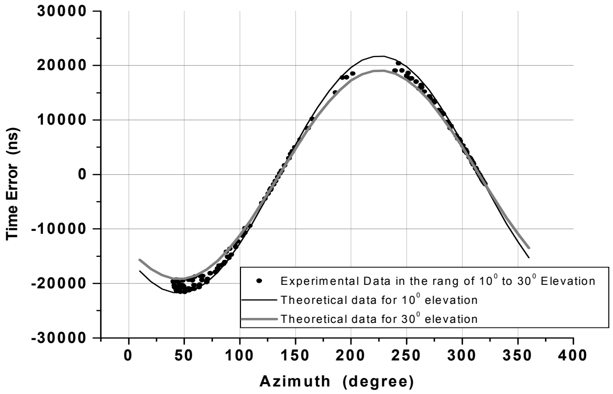

Fig. 5. Experimental Observations of Time Error within the elevation range of satellite between 10^0 to 30^0 with theoretical values in solid line for elevation 10^0 & 30^0 for given position error.

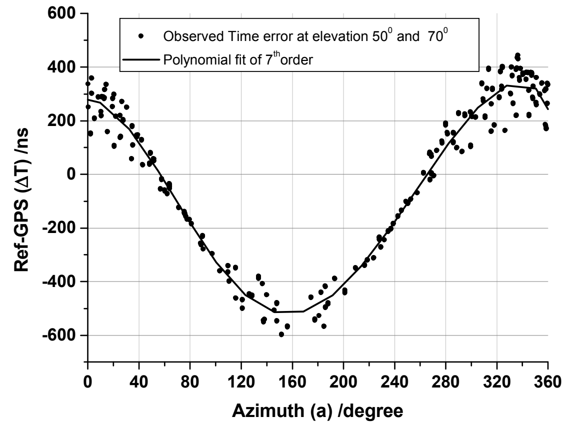

Fig. 6. Experimental data for the location whose coordinates are to be corrected. the polynomial fit-line approximates the curve equivalent to equation 13.

Out of these data, "Ref-GPS" in 10^{th} column which corresponds to ΔT and theelevation (e) and the azimuth (a) of the satellite in 6^{th} and 7^{th} column respectively are only of interest in the present study.

To start with let us concentrate on observed data for a particular value of the elevation angle (60^o in this case). Consequently for such a case, the variation of "Ref-GPS" (ΔT) with the azimuth (a) would be governed by a simpler version of equation (5) with elevation (e) value of 60^o as given below.

$$\Delta T = \frac{1}{30}[0.5E_x.Cos(a) + 0.5E_y.Sin(a) + 0.866E_z] \qquad (13)$$

But it was not possible to get sufficient number of observations for the particular elevation angle of 60^o in the observed set of data. So to get enough data "Ref-GPS" (ΔT) has been sorted for the elevation angles in the range of 50^o to 70^o, mid value between them being 60^o. These data have been shown in Fig. 6. It is very difficult to fit a sinusoidal curve equivalent to equation (13) on the data of Fig. 6. So a 7^{th} order least square polynomial fit has been drawn as shown by solid line in Fig.6. The fit-line may be assumed to approximate the curve that may be generated from equation (13). Any three points in the linear portion of the fit curve (in the region between 242^o and 284^o) are chosen, so that the corresponding values of "azimuth" and "Ref-GPS" are used in equation (13) to form three linear equations.

$$-6214.0488 = [-0.248E_x - 0.43415E_y. + 0.8660E_z] \qquad (14)$$

$$-1409.28 = [-0.0664E_x - 0.49555E_y. + 0.8660E_z] \qquad (15)$$

$$-2995.7571 = [0.12495E_x - 0.4841E_y. + 0.8660E_z] \qquad (16)$$

These equation (14-16) have been used to find the values of E_x, E_y and E_z. By using the values of E_x E_y and E_z in equations (6), (7), and (8) the values of Δx_n, Δy_n and Δz_n have been evaluated by solving three equations. Subscript "n" represents the solutions for this proposed new technique. This exercise of finding Δx_n, Δy_n and Δz_n has been repeated using the linear region between 41^o and 85^o of Fig. 6.

Similar way, two more sets of Δx_n, Δy_n and Δz_n have been found by sorting the observed data for 20^o elevation. All these values have been given in Table II.

To validate these results, the coordinates of this location were later determined by a precise geodetic receiver [6] and by using post-dated precise parameters from IGS network. The values of Δx_p, Δy_p and Δz_p (subscript "p" refers to the precise technique) thus determined have also been included in Table II for direct assessment of the accuracy of the proposed technique. The Table II depicts that the accuracies of the proposed new technique lie well within 20 cm.

TABLE I
TYPICAL SAMPLE OF DATA AVAILABLE FROM A GPS TIMING RECEIVER

PRN	CL	MJD	STTIME	TRKL	ELV	AZTH	REFSV	SRSV	REFGPS
			hhmmss	s	.1dg	.1dg	.1ns	.1ps/s	.1ns
16	FF	54434	202200	780	285	1616	-1263547	-433	7812
14	FF	54434	202200	780	255	667	3163618	801	-94
11	FF	54434	203800	780	230	2585	-263976	841	-915
20	FF	54434	203800	780	326	3184	-1259215	632	-6015
22	FF	54434	203800	780	142	1361	-2034671	327	7577
31	FF	54434	203800	780	575	277	-5652	-531	-2036
1	FF	54434	203800	780	448	449	-1699057	689	-1773
14	FF	54434	203800	780	219	735	3164435	910	749

TABLE II

ACCURACY OF THE PROPOSED TECHNIQUE FOR THE REFINEMENT OF POSITION COORDINATES

Δx_n	Δx_p	Δx_n-Δx_p	Δy_n	Δy_p	Δy_n-Δy_p	Δz_n	Δz_p	Δz_n-Δz_p
5003.42	4989.36	14.06	-17008.30	-16994.82	-13.48	18370.46	18371.44	-0.98
4998.12	4989.36	8.76	-16991.40	-16994.82	3.42	18371.75	18371.44	0.31
5000.67	4989.36	11.31	-17005.23	-16994.82	-10.41	18372.02	18371.44	0.58
5004.70	4989.36	15.34	-17009.16	-16994.82	-14.34	18371.97	18371.44	0.53

VI. CONCLUDING REMARKS

For time transfer through single satellite GPS technique, the precise knoweldge of position coordinate is important. The relation between the position error and the corresponding calculated error in the Time of the GPS receiver has been derived. Time of the GPS receiver caused by the error in position coordinates largely depends on the position of the satellite indicated by the respective elevation and azimuth of the satellite. Observed data obtained through a specially planned experiment has been found to match the derived relation excellently.

Application of the observed data on the analytical expression leads to the precise determination of the position coordinates within an accuracy of 20 cms.

The proposed technique is much simpler than the very elaborate method with the help of the geodetic receiver and the post dated IGS products. The accuracy of position coordinates that can be achieved by this technique is obviously limited by the noise in time error data.

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