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Note: Measuring capacitance and inductance of a helical resonator and improving its quality factor by mutual inductance alteration

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Narrow bandwidth and high voltage radio frequency (RF) is an essential requirement for stable confinement of ions within a RF trap and helical resonators are commonly used for that purpose. Effective capacitance and inductance of a helical resonator are estimated by measuring resonant frequencies for different external loads. Load capacitance of an ion trap can be estimated from this method and a resonator can be constructed for desired resonant frequency. We demonstrate a very simple method to achieve higher Q-factor of a resonator by optimizing mutual separation between the primary antenna and helical coil. We also formulate a set of analytical equations for calculating overall inductance, resistance, and Q-factor of a loaded helical resonator. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4919910]

Ions confined within radio frequency (RF) trap have immense application in quantum information processing,[1,2] quantum simulation,[3,4] and frequency standards.[5,6] RF driven ion traps require narrow bandwidth and high amplitude (~kV) at high frequency in order to achieve large trapping potential, i.e., longer trapping lifetime.[7] Most commonly high voltage and narrow bandwidth RF has been delivered to the trap electrodes by employing a resonant circuit with a quarter wave resonator or helical resonator.[8,9] Such a resonator acts as narrow bandwidth filter with centre frequency of several megahertz and it allows impedance matching between a RF generator and the ion trap, enabling delivery of high voltages into the ion trap. The key components of a helical resonator are a cylindrical copper tube as a shield, a helical coil, and a primary antenna for coupling RF signal to the resonator.[9] The goal of our experiment is to deliver a high voltage (~kV) RF to an ion trap within a frequency range of 20-30 MHz.[10] Macalpine and Schildknecht[11] provided the design chart for helical resonator and described in detail how its performances depend on the different parameters of the resonator. They also provided a set of analytical formulas for estimating resonant frequency (f_0) and Q-factor (Q) of the resonator at unloaded condition.[11] We have built a helical resonator with copper of shield diameter 120 mm, helical coil diameter 60 mm, and helical coil height 95 mm with 8.5 helical turns. The primary antenna is also made with a 2 mm thick copper wire and it has 6 turns of diameter 37 mm and the overall height is 37 mm.

As per the empirical formulas, the resonant frequency and quality factor of the resonator are expected to be $f_0 \sim 50$ MHz and $Q \sim 1500$ at unloaded condition. However, we need to compute and account for changes in the f_0 and Q of the resonator when it is connected to the ion trap due to its capacitive load. The f_0 and Q of the resonator are measured by monitoring the reflected signal from the resonator with a unidirectional coupler.[9] The measured resonant frequency at unloaded condition is $f_0 = 38.75$ (± 0.5) MHz with $Q = 561$ (± 20), those are quite different compared to the calculated values. In the calculation, only the isolated operation of the resonator is considered, where the capacitive load of the conductors for delivering RF to the trap has not been considered. Once the ion trap is connected with the resonator, its f_0 and Q change due to the load capacitance of the trap and its connectors.[9]

The resonant frequency of a resonator depends on its capacitance and inductance. We have adopted a very simple technique for estimating equivalent capacitance and inductance of the helical resonator by measuring its resonant frequencies for different external load capacitances. Fig. 1 shows the variation of resonant frequency of the helical resonator for different load capacitances. Resonant frequency at loaded condition can also be calculated utilizing lumped element circuit model,[12] where the external load capacitance (C_L) considered to be connected parallel to the total equivalent capacitance (C) of the resonator. If f_0 and $f_0^/$ are the resonant frequencies of the resonator at unloaded condition and with a load capacitance C_L, they should follow the relationship $f_0 = \frac{1}{2\pi}\sqrt{1/LC}$ and $f_0^/ = \frac{1}{2\pi}\sqrt{1/L(C+C_L)}$ or $C = \left(C_L \Big/ \{(\frac{f_0}{f_0^/})^2 - 1\}\right) = \left(C_L \Big/ \delta f^2\right)$, where $\delta f^2 = \{(\frac{f_0}{f_0^/})^2 - 1\}$.

Measuring δf^2 for different load capacitances (C_L), the equivalent capacitance (C) of the resonator can be calculated. Fig. 2(a) shows the linear dependence of δf^2 with C_L and the calculated C of the resonator to be 8.73 (± 0.4) pF. The relation between $f_0^/$ and C_L can be expressed as

$$f_0^{/2} = 1/4\pi^2 L(C + C_L) = 1/4\pi^2 LC^/, \text{ where } C^/ = C + C_L.$$

Now studying the variation of $f_0^{/2}$ with C_L, the inductance of the helical resonator can be calculated. Fig. 2(b) shows the variation of $f_0^{/2}$ with the inverse of total capacitance ($C^/$) and the calculated inductance (L) of the helical resonator to be 1.93 (± 0.08) μH. Considering $C = 8.73$ pF and $L = 1.93$ μH, we have simulated the resonant frequencies for a wide range of external load capacitance and it is in good agreement with the experimentally measured values (Fig. 1). Load capacitance of an ion trap can also be estimated from the loaded resonant

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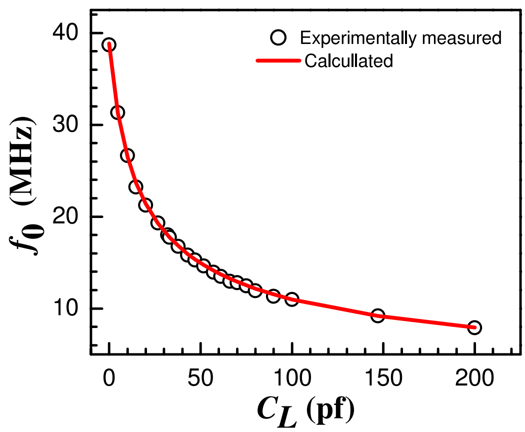

FIG. 1. Variation of resonant frequency (f_0) with load capacitance (C_L).

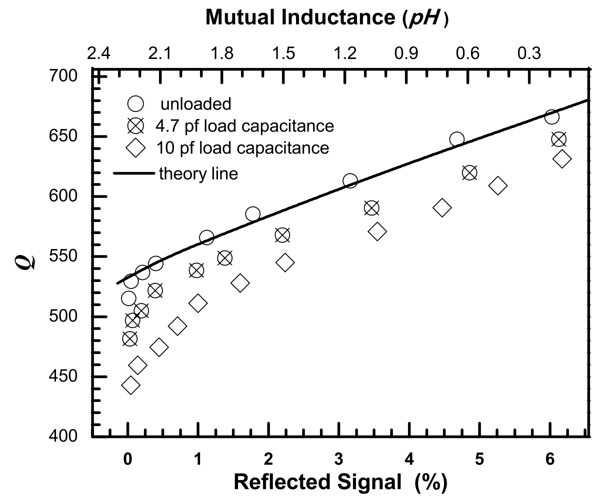

FIG. 3. Variation of Q with reflected signal for different load capacitances. The solid line depicting the variation of Q (theoretically calculated) with mutual inductances.

frequency and a resonator can be constructed for a desired resonant frequency. During the measurement of Q, care should be taken for efficient coupling of RF power from source to the resonator. The small loop primary antenna is placed very near to the helical coil for low impedance coupling of RF power. Separation between the two coils is utilized for adjusting the power transfer efficiency from the RF source to the resonator. Position of the primary antenna is optimized for the best coupling, i.e., with negligible amount of reflected RF power from the resonator. The reflected power increases when the primary antenna is moved from its optimized position. The most significant observation is that not only the reflected power varies with the position of the primary antenna, but also the Q has strong dependence on the mutual separation between the two coils; Q increases with the increase of mutual separation between the two coils. The power transfer efficiency from RF source to the resonator depends on their impedance matching. So, change in coupling between the coils affects the overall impedance of the resonator, which finally results into change in the power transfer efficiency between the source and the resonator. The overall impedance (Z) of a helical resonator can be expressed as[9]

$$Z = i\omega L_p + \omega^2 M^2/(i\omega L_h + Z_l),$$

where L_p and L_h are the self-inductance of the primary antenna and the helical coil, respectively, and Z_l is the load impedance. M is the mutual inductance between the two coils, which depends on their mutual separation and can be expressed as $M = \kappa\sqrt{L_p L_h}$, where κ is the coupling parameter. Overall impedance of a resonator can either be changed by modifying the self-inductance of the primary antenna or by changing the coupling, i.e., the mutual separation between the two coils. Siverns *et al.* showed that the overall impedance of a resonator can be altered by changing the number of turns and pitch of the primary antenna and better impedance matching between the source and the resonator can be achieved.[9] In this work, we intended to demonstrate that the Q-factor of a resonator can be improved by optimization of mutual separation between the primary antenna and helical coils.

Once the impedance matching between the source and the resonator is achieved, hardly there is any reflected signal from the resonator. Displacement of the primary antenna from its optimized position causes impedance mismatch and the reflected power starts appearing. Accurate measurement of the separation between these two coils is extremely difficult, since they are placed inside the copper shield. Measuring the reflected power from the resonator, it is possible to monitor the separation between the two coils, as the reflected power is correlated with the mutual separation between the two coils. Fig. 3 depicts the dependencies of Q of the resonator with the reflected power. The Q increases with the increase of reflected power, i.e., with the increase of separation between primary antenna and the helical coil. Similar studies have been repeated for different load capacitances and every time it has been observed that Q increases with the increase of separation between the two coils (Fig. 3). The Q of the

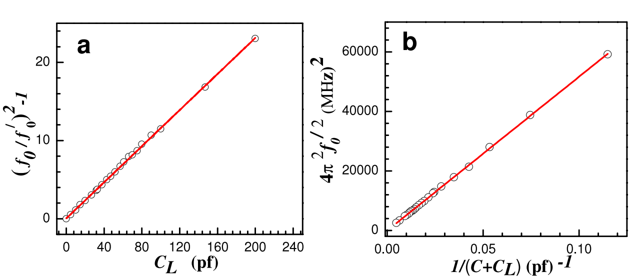

FIG. 2. (a) Variation of δf^2 with load capacitance and (b) linear dependence of the $f_0'^2$ with inverse of total capacitance (C').

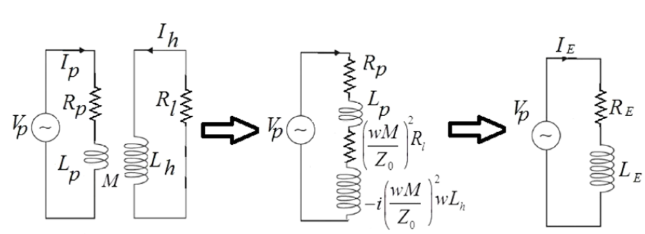

FIG. 4. (a) Transformer model of the helical resonator. (b) Equivalent circuit of the resonator, applying transformer model. (c) Simplified equivalent circuit with effective inductance (L_E) and resistance (R_E).

resonator increases up to 25% at loaded condition for an initial displacement ∼2 mm of the primary antenna, whereas the reflected RF signal is increased by ∼2%–3% only. Further movement of the primary antenna does not show similar rate of increment of the Q with respect to the reflected signal. An overall displacement ∼5 mm of the primary antenna from its optimized position causes an increase in the reflected signal ∼5% but the Q-factor increases up to 35%. Further movement of the primary antenna does not show significant improvement of Q but the coupling between the two coils becomes much weaker and causes a sharp increase in the reflected signal strength. Higher values of reflected RF power may cause damage to the source, particularly when it operates at high power (∼Watts). So, we have restricted our studies with reflected power up to 6% of the incident power. Our present study clearly shows that the Q of the resonator at loaded condition changes substantially with the movement of the primary antenna and the rate of change is maximum when the reflected power stays within 2% of the incident power.

To understand the above mechanism, i.e., the dependencies of Q with the separation between the primary antenna and the helical coil, we have modelled the resonator along with the primary antenna as air core transformer.[13,14] The primary antenna, electrically connected with the RF source, is considered to be transformer primary coil and the helical coil as the secondary coil (Fig. 4(a)). L_p and R_p are the inductance and resistance of the primary antenna, and V_p and I_p are the voltage and current across it. L_h and R_h are the inductance and resistance of the helical coil coupled to the primary antenna with an impedance of Z_0 and I_h is the inductive current across the helical coil. Applying transformer model and considering contribution from individual components, an equivalent circuit of the resonator has been drawn (Fig. 4(b)). The coupled circuits are transferred into a simple form (Fig. 4(c)) by circuit analysis where the secondary elements are written in terms of primary circuit current. Utilizing the simplified equivalent circuit, effective inductance (L_E) and resistance (R_E) of the resonator have been estimated,[14]

$$L_E = \left\{ L_p - M^2\omega^2 \left(\frac{L_h}{R_l^2 + \omega^2 L_h^2} \right) \right\}$$

and

$$R_E = \left\{ R_p + M^2\omega^2 \left(\frac{R_l}{R_l^2 + \omega^2 L_h^2} \right) \right\}.$$

The effective quality factor Q_E of the resonator can be expressed as $Q_E = \frac{1}{R_E}\sqrt{\frac{L_E}{C}}$ and substituting R_E and L_E we get

$$Q_E = \frac{1}{\sqrt{C}} \left[\left\{ L_p - M^2\omega^2 \left(\frac{L_h}{R_l^2 + \omega^2 L_h^2} \right) \right\} \Big/ \left\{ R_p + M^2\omega^2 \left(\frac{R_l}{R_l^2 + \omega^2 L_h^2} \right) \right\}^2 \right]^{1/2}.$$

The above equations show the dependence of effective Q-factor with mutual inductances. A theory line is plotted in Fig. 3 to depict the variation of Q with the mutual inductances between the primary antenna and the helical coils. Coupling between the primary antenna and helical coils becomes weaker with the increase of separation between them and causes decrease of M. Any decrease of the M causes an increase in the effective inductance, whereas the effective resistance decreases with the decrease of M and the overall Q_E of the system increases with the decrease of M. The above analytical formula of Q_E explains the experimental observation about the increase of Q of the helical resonator with the increase of separation between the primary antenna and the helical coil. One of the setbacks of this technique about improving Q by reducing the mutual inductance is part of the incident power is reflected back from the resonator due to weaker coupling between the two coils. The prime objective of achieving very high Q of a helical resonator is delivering high voltage RF to the ion trap. The rms voltage across the trap electrode depends on the input power P and Q of the resonator[9] and can be expressed as $V_{rms} = \zeta\sqrt{PQ}$, where $\zeta = (L/C)^{1/4}$ is the trap parameter. Helical resonators with higher Q have better efficiency of converting its input power into high voltage output, i.e., higher Q of a resonator provides higher output voltages per input power.

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