

Experimental study of the relation between the degrees of coherence in space-time and space-frequency domain

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Abstract: We present an experimental study showing the effect of the change in the bandwidth of light on the magnitude of both the complex degree of coherence and the spectral degree of coherence at a pair of points in the cross-section of a beam. A variable bandwidth source with a Young's interferometer is utilized to produce the interference fringes. We also report for the first time that if the field is quasi-monochromatic or sufficiently narrowband, the elements of both the beam coherence polarization matrix and the cross-spectral density matrix, normalized to intensities (spectral densities) at the two points possess identical values.

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OCIS Codes: (030:1640) Coherence; (260:3160) Interference; (260:5430) Polarization.

References and links

1. L. Mandel, and E. Wolf, "Coherence properties of optical fields," *Rev. Mod. Phys.* **37**(2), 231–287 (1965).
2. M. Born, and E. Wolf, *Principles of Optics*, 7th ed. (Cambridge: Cambridge University Press, 1999), chapter 10.
3. L. Mandel, and E. Wolf, "Spectral coherence and the concept of cross-spectral purity," *J. Opt. Soc. Am.* **66**(6), 529–535 (1976).
4. E. Wolf, "New theory of partial coherence in the space frequency domain. Part I: Spectra and cross spectra of steady state sources," *J. Opt. Soc. Am.* **72**(3), 343–351 (1982).
5. E. Wolf, "New theory of partial coherence in the space frequency domain. Part II: Steady-state fields and higher-order correlations," *J. Opt. Soc. Am. A* **3**(1), 76–85 (1986).
6. L. Mandel, and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press, 1995), chapters 4 and 7.
7. E. Wolf, "Young's interference fringes with narrow-band light," *Opt. Lett.* **8**(5), 250–252 (1983).
8. A. T. Friberg, and E. Wolf, "Relationships between the complex degrees of coherence in the space-time and in the space-frequency domains," *Opt. Lett.* **20**(6), 623–625 (1995).
9. D. F. V. James, and E. Wolf, "Some new aspects of Young's interference experiment," *Phys. Lett. A* **157**(1), 6–10 (1991).
10. L. Basano, P. Ottonello, G. Rottigni, and M. Vicari, "Spatial and temporal coherence of filtered thermal light," *Appl. Opt.* **42**(31), 6239–6244 (2003).
11. B. Kanseri, and H. C. Kandpal, "Experimental observation of invariance of spectral degree of coherence with change in bandwidth of light," *Opt. Lett.* **35**(1), 70–72 (2010).
12. E. Wolf, *Introduction to Theory of Coherence and Polarization of Light* (Cambridge: Cambridge University Press, 2007), chapters 3 and 4.
13. J. C. Ricklin, and F. M. Davidson, "Atmospheric turbulence effects on a partially coherent Gaussian beam: implications for free-space laser communication," *J. Opt. Soc. Am. A* **19**(9), 1794–1802 (2002).
14. F. Gori, "Matrix treatment for partially polarized, partially coherent beams," *Opt. Lett.* **23**(4), 241–243 (1998).
15. F. Gori, M. Santarsiero, S. Vicalvi, R. Borghi, and G. Guattari, "Beam coherence-polarization matrix," *J. Opt. A: Pure App. Opt.* **7**(5), 941–951 (1998).
16. B. Kanseri, S. Rath, and H. C. Kandpal, "Determination of the beam coherence polarization matrix of a random electromagnetic beam," *IEEE J. Quantum Electron.* **45**(9), 1163–1167 (2009).
17. E. Wolf, "Unified theory of coherence and polarization of random electromagnetic beams," *Phys. Lett. A* **312**(5–6), 263–267 (2003).
18. H. Roychowdhury, and E. Wolf, "Determination of the electric cross-spectral density matrix of a random electromagnetic beam," *Opt. Commun.* **226**(1–6), 57–60 (2003).
19. B. Kanseri, S. Rath, and H. C. Kandpal, "Determination of the amplitude and the phase of the elements of electric cross-spectral density matrix by spectral measurements," *Opt. Commun.* **282**(15), 3059–3062 (2009).

20. B. Kanseri, and H. C. Kandpal, "Experimental determination of electric cross-spectral density matrix and generalized Stokes parameters for a laser beam," *Opt. Lett.* **33**(20), 2410 (2008).
21. G. P. Agrawal, and E. Wolf, "propagation-induced polarization changes in partially coherent optical beams," *J. Opt. Soc. Am. A* **17**(11), 2019–2023 (2000).
22. F. Gori, M. Santarsiero, R. Borghi, and G. Piquero, "Use of the van Cittert-Zernike theorem for partially polarized sources," *Opt. Lett.* **25**(17), 1291–1293 (2000).
23. E. Wolf, "Correlation-induced changes in the degree of polarization, the degree of coherence, and the spectrum of random electromagnetic beams on propagation," *Opt. Lett.* **28**(13), 1078–1080 (2003).
24. O. Korotkova, M. Salem, and E. Wolf, "The far zone behaviour of the degree of polarization of electromagnetic beams propagating through atmospheric turbulence," *Opt. Commun.* **233**(4-6), 225–230 (2004).
25. J. Tervo, T. Setälä, and A. T. Friberg, "Theory of partially coherent electromagnetic fields in the space–frequency domain," *J. Opt. Soc. Am. A* **21**(11), 2205 (2004).
26. O. Korotkova, and E. Wolf, "Spectral degree of coherence of a random three-dimensional electromagnetic field," *J. Opt. Soc. Am. A* **21**(12), 2382–2385 (2004).
27. J. Ellis, and A. Dogariu, "Complex degree of mutual polarization," *Opt. Lett.* **29**(6), 536–538 (2004).
28. P. Réfrégier, and F. Goudail, "Invariant degrees of coherence of partially polarized light," *Opt. Express* **13**(16), 6051–6060 (2005).
29. H. T. Eyyuboğlu, Y. Baykal, and Y. Cai, "Complex degree of coherence for partially coherent general beams in atmospheric turbulence," *J. Opt. Soc. Am. A* **24**(9), 2891 (2007).
30. Y. Cai, O. Korotkova, H. T. Eyyuboğlu, and Y. Baykal, "Active laser radar systems with stochastic electromagnetic beams in turbulent atmosphere," *Opt. Express* **16**(20), 15834–15846 (2008).
31. M. Salem, and E. Wolf, "Coherence-induced polarization changes in light beams," *Opt. Lett.* **33**(11), 1180–1182 (2008).
32. P. Réfrégier, and A. Roueff, "Coherence polarization filtering and relation with intrinsic degrees of coherence," *Opt. Lett.* **31**(9), 1175–1177 (2006).
33. M. Salem, and G. P. Agrawal, "Effects of coherence and polarization on the coupling of stochastic electromagnetic beams into optical fibers," *J. Opt. Soc. Am. A* **26**(11), 2452–2458 (2009).
34. S. N. Volkov, D. F. V. James, T. Shirai, and E. Wolf, "Intensity fluctuations and the degree of cross-polarization in stochastic electromagnetic beams," *J. Opt. A, Pure Appl. Opt.* **10**(5), 055001 (2008).
35. E. Fortin, "Direct demonstration of the Fresnel-Arago laws," *Am. J. Phys.* **38**(7), 917–918 (1970).

1. Introduction

Partial coherence properties of the scalar optical fields, both in the space–time and in the space-frequency domain have continuously been a subject of great interest in the last century. The degree of coherence of optical fields, both in the space-time [1, 2] and in the space-frequency domain [3–6] have proved to be a strong tool for characterizing various correlation properties of optical sources. Later, several theoretical studies [7–9] and more recently, some experimental studies [10, 11] showed that the spectral degree of coherence is a natural property of optical fields that remains unchanged in the process of filtering. Moreover, it was also predicted that the magnitude of the complex degree of coherence does not approach to unity when the frequency pass-band of light is decreased. For the central fringe, the quantity attains its maximum value, which is equal to the absolute value of the spectral degree of coherence of the light [7, 8]. The space-time and the space-frequency treatments of coherence functions have found numerous applications in various fields [6, 12]. It was shown theoretically not long ago that if the field is strictly monochromatic or sufficiently narrowband, then both these characterizations yield identical results [13].

Recently, we have given an experimental method to construct a variable band-width source using a variable slit monochromator [11]. Using the secondary source, it was shown that the spectral degree of coherence remains unchanged on filtering the light. However, this experimental study is incomplete unless the dependence of the complex degree of coherence on change in the spectral-width of light and its relation with the spectral degree of coherence is made clear. Therefore, a vivid observation of these effects is essential. Also in recent years, the coherence properties of the electromagnetic beams have been associated with their polarization properties. For vectorial fields, matrices characterizing the coherence and the polarization properties simultaneously are called the beam coherence polarization (BCP) matrix [14–16] and the cross-spectral density (CSD) matrix [17–20] in the space-time and in the space-frequency domain, respectively. Using these matrices, several studies on various propagation dependent properties have been made in the last decade [21–34]. Though these

matrices have been proposed nearly one decade back yet the equivalence condition for the elements of these matrices has not been studied till now. Therefore, in this respect, it would be timely and interesting to investigate the usefulness of the scalar field based results for the vectorial (electromagnetic) fields, and find the condition for which the BCP and the CSD matrices could have identical elements.

The main objective of this paper is to study relationship between the degrees of coherence in space-time and in space-frequency domain, experimentally. Instead of using separate filters having altered bandwidth (as in case of [10]), which might have slight dissimilarity in construction (in practice), we have used a single source which can be manipulated to produce light fields of desired bandwidths. Moreover, narrowband (interference) filters are very much susceptible to alignment, which will not be a case with present study. We have constructed a variable bandwidth source (secondary) using a variable slit monochromator and a continuous spectrum lamp [11]. Using a double-slit, interference fringes were obtained at the observation plane and their visibility was determined for different bandwidths of the impinging light. It was observed that the time domain visibility increases with the decrease in the bandwidth and approaches to the maximum value limited by the spectral visibility of the light. We also show that the results obtained using the scalar treatments in section 4 of this paper, have importance in connection with the recently formulated polarization and coherence theories. The experimentally verified relation between the degrees of coherence is used to find the condition of equivalence for the normalized elements of the BCP and the CSD matrices. Using the same experimental setup, it is shown that in the quasi-monochromatic limit, the normalized elements of the two matrices possess identical values.

2. Relation between the complex degrees of coherence

In space-frequency treatment of the partially coherent light, the spectral degree of coherence is described mathematically as [6]

$$\mu(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{W(\mathbf{r}_1, \mathbf{r}_2, \omega)}{[W(\mathbf{r}_1, \mathbf{r}_1, \omega)W(\mathbf{r}_2, \mathbf{r}_2, \omega)]^{1/2}}, \quad (1)$$

where $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is the cross-spectral density of light and $W(\mathbf{r}_i, \mathbf{r}_i, \omega)$ (for $i=1, 2$) are the spectral densities at the two points. Similarly, the complex degree of coherence that contains correlation information at a pair of points in space-time domain is given by [2]

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)}{[\Gamma(\mathbf{r}_1, \mathbf{r}_1, 0)\Gamma(\mathbf{r}_2, \mathbf{r}_2, 0)]^{1/2}}, \quad (2)$$

where $\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)$ represents the mutual coherence function, τ is the time difference and $\Gamma(\mathbf{r}_i, \mathbf{r}_i, 0)$ (for $i=1, 2$) are the intensities at the respective points in the source plane.

As shown in [8], a relation could be established between the two degrees of coherence which is given by

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \int_0^\infty [s(\mathbf{r}_1, \omega)]^{1/2} [s(\mathbf{r}_2, \omega)]^{1/2} \mu(\mathbf{r}_1, \mathbf{r}_2, \omega) \exp(-i\omega\tau) d\omega, \quad (3)$$

where the normalized spectra of the field at the two points are given by

$$s(\mathbf{r}_i, \omega) = \frac{S(\mathbf{r}_i, \omega)}{\int S(\mathbf{r}_i, \omega) d\omega}, \quad \text{for } i=1, 2. \quad (4)$$

In Eq. (4) quantity $S(\mathbf{r}_1, \omega) = W(\mathbf{r}_1, \mathbf{r}_1, \omega)$ is the spectral density of light at point \mathbf{r}_1 [see Eq. (1)]. As a special case, if $s(\mathbf{r}_1, \omega) = s(\mathbf{r}_2, \omega) = s(\mathbf{r}, \omega)$, and the light is quasi-monochromatic, i.e. its effective frequency range is much smaller than the mean frequency ($\Delta\omega \ll \omega_0$), then the spectral degree of coherence $\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)$ does not depend on ω over the bandwidth $\Delta\omega$ of light. In these conditions, taking $\tau = 0$ readily gives [8],

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, 0) = \mu(\mathbf{r}_1, \mathbf{r}_2, \omega), \quad (5)$$

i.e. the equal time complex degree of coherence is equal to the spectral degree of coherence of light. The directly measurable quantity in this respect is the absolute value of the degree of coherence (or the visibility of the interference fringes). Taking modulus on both sides of Eq. (5), we get

$$|\gamma(\mathbf{r}_1, \mathbf{r}_2, 0)| = |\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)|. \quad (6)$$

It is worth to emphasize here that the condition imposed to arrive on Eq. (5) is that the effective frequency range should be smaller than the central frequency of light. One way to obtain this condition experimentally is to put identical filters having sufficiently narrow pass-band in front of both the slits in the Young's interference experiment [7, 10]. Another way to achieve this feat may be to use a variable bandwidth source, and such a secondary source was constructed using a variable slit monochromator as reported in [11].

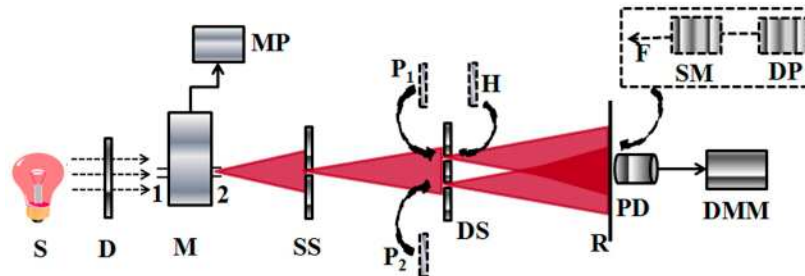


Fig. 1. Schematics of the experimental setup. S Tungsten halogen lamp, D diffuser, M monochromator with microprocessor (MP) control (1, 2 are the entrance and exit slits of the monochromator, respectively), SS single slit, P polarizer, DS double slit, H half wave plate, R observation plane, PD photodiode, DMM digital multimeter, F fiber, SM spectrometer and DP data processor.

3. Experimental setup

A tungsten-halogen lamp S (Mazda, colour temperature = 3200 K) with diffused outer glass jacket was operated at 500 W using a regulated dc power supply (Heinzinger, stability 1 part of 10^4) as shown in Fig. 1. The light coming out of the uniformly illuminating, polychromatic, continuous spectrum source was passed through a microprocessor (MP) controlled monochromator M (CVI, Digikrom) having variable entrance and exit slits. As shown in Fig. 1, a single-slit SS (slit width 0.4 mm) was placed at a distance 40 cm from the exit slit of the monochromator to control the coherence developed at the double-slit plane [11]. The beam emerging from SS illuminated a double-slit DS having rectangular slits with slit width 0.15 mm and slit separation 0.25 mm and was placed 70 cm away from SS. The beams emerging from the double-slit interfered and the fringes were obtained at a distance of 30 cm from DS (see Fig. 2). The time-domain measurements were made using a silicon photodetector PD (Perkin Elmer, sensing area $120 \mu\text{m}^2$) interfaced with a digital multimeter DMM (Keithley, 2000 model). The spectral measurements were carried out using a fibre (F)

coupled spectrometer SM (Photon Control, SPM002, spectral bandwidth \approx 0.5 nm) interfaced with a personal computer DP (see Fig. 1).

4. Results and discussion

In this experimental study, the central wavelength of the broadband, variable bandwidth, secondary source was chosen 670 nm because of maximum detection sensitivity of the photodetector at this wavelength. Using the microprocessor control, the bandwidth of the emerging light was changed from 4.6 nm to 0.6 nm, as shown in our previous work (see Fig. 2 in [11]). The interference fringes obtained for various bandwidths of light are shown in Fig. 2. In our case, the effective bandwidth of the light $\Delta\omega$ was much smaller than the peak frequency ω of light (3×10^{12} Hz \ll 4.8×10^{14} Hz) and therefore, the spectral degree of coherence $\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)$ for the pair of points could be assumed to be substantially constant over the effective bandwidth of light.

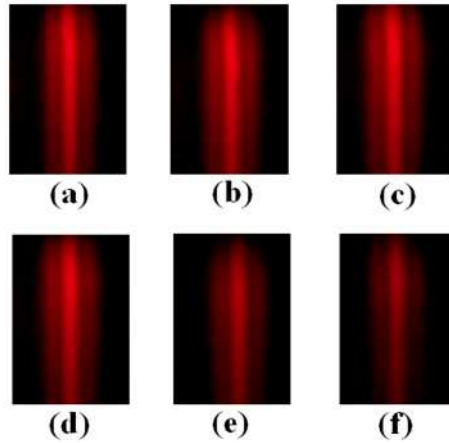


Fig. 2. Photograph of the interference fringes obtained at plane R. The entrance and the exit slits of the monochromator were opened for (a) 2, (b) 1.6, (c) 1.2, (d) 0.8, (e) 0.4 and (f) 0.1 mm.

The spectral visibility could be determined by taking intensity (spectral) measurements horizontally across the fringe pattern and using the relation [6]

$$|\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)| = \frac{S_{\max}(\mathbf{r}, \omega) - S_{\min}(\mathbf{r}, \omega)}{S_{\max}(\mathbf{r}, \omega) + S_{\min}(\mathbf{r}, \omega)}, \quad (7)$$

where $S_{\max}(\mathbf{r}, \omega)$ and $S_{\min}(\mathbf{r}, \omega)$ are respectively the maximum and the minimum values of spectral density around the observation point \mathbf{r} in the central fringe. Similarly, the visibility of fringes $|\gamma(\mathbf{r}_1, \mathbf{r}_2, 0)|$ could be determined using Eq. (7), replacing spectral density S by intensity I [2, 6]. The magnitude of the spectral degree of coherence $|\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)|$ was measured for different values of the bandwidth and is shown in Fig. 3(a). The graph shows that within experimental uncertainty, this quantity remains constant throughout the bandwidth range. *Thus the absolute value of the spectral degree of coherence remains unaffected by the change in the bandwidth of light* [11].

Figure 3(b) shows the behaviour of the magnitude of the complex degree of coherence with change in the bandwidth of light. The fringe visibilities of the central (upper curve) and the off central fringes, i.e. either the side of the central fringe (lower curve) decrease for higher bandwidths. It also shows that, for smaller frequency passband, the visibility (time-

domain) approaches to the spectral visibility of light, which is its maximum (limiting) value (see Fig. 3). This infers that *the maximum visibility of the interference fringes formed by the filtered light will not, in general, tend to unity as the pass-bands of the filters decrease* [7, 8]. A careful look on the interference patterns in Fig. 2 also reveals that the visibility of the central fringe is more than that of the off central fringes. However, the change in visibility from (a) to (f) in Fig. 2 could not be judged so accurately by naked human eye due to the decrease in overall intensity (throughput) of light with decrease in bandwidth.

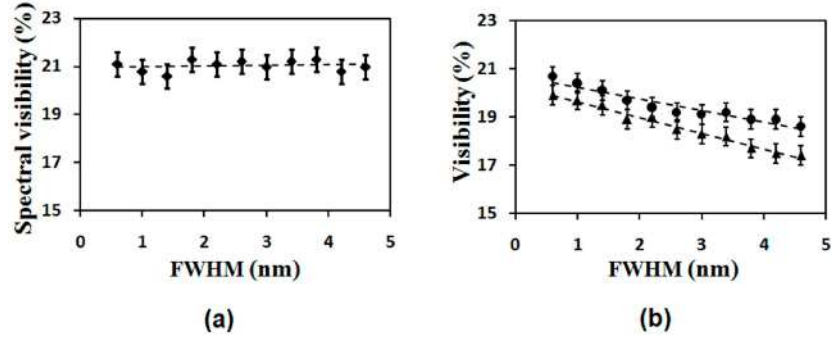


Fig. 3. (a) Behaviour of the magnitude of the spectral degree of coherence (spectral visibility) with the change in the bandwidth of the light. (b) The change in the absolute value of the degree of coherence (visibility) with bandwidth of light, measured for the central fringe (shown by dots) and for either side of the central fringe (shown by triangles). The dotted line in (a) and (b) shows the trend line plotted as linear fit. The error bars show the uncertainty in the measurements calculated at 95% confidence interval.

5. Application for the elements of BCP and CSD matrices

The elements of the BCP matrix can be determined using the relation [15, 16]

$$J_{ij}(\mathbf{r}_1, \mathbf{r}_2, z) = \sqrt{I_i(\mathbf{r}_1, z)} \sqrt{I_j(\mathbf{r}_2, z)} \gamma_{ij}(\mathbf{r}_1, \mathbf{r}_2, z), \quad (8)$$

where $\gamma_{ij}(\mathbf{r}_1, \mathbf{r}_2, z)$ for $i = x, y; j = x, y$ are the values of the complex degrees of coherence (equal time degree of coherence, $\tau = 0$) and $I_i(\mathbf{r}_1, z), I_j(\mathbf{r}_2, z)$ are the field intensities for different polarized components of the beam [14]. For the sake of convenience, we omit the term z (direction of propagation) and use $\tau = 0$ in the equation. Thus we can write Eq. (8) as

$$\frac{J_{ij}(\mathbf{r}_1, \mathbf{r}_2, 0)}{\sqrt{I_i(\mathbf{r}_1)} \sqrt{I_j(\mathbf{r}_2)}} = \gamma_{ij}(\mathbf{r}_1, \mathbf{r}_2, 0). \quad (9)$$

A similar equation could be written for the elements of the CSD matrix as [18, 20]

$$\frac{W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega)}{\sqrt{S_i(\mathbf{r}_1, \omega)} \sqrt{S_j(\mathbf{r}_2, \omega)}} = \mu_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega), \quad (10)$$

where $\mu_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ for $i = x, y; j = x, y$ are the elements of spectral degrees of coherence for different polarized components of the light beam [34].

It has been predicted by Riklin and Davidson [13] that, if the field is strictly monochromatic or sufficiently narrowband, so that

$$\frac{|\mathbf{p}_2 - \mathbf{p}_1|}{c} \ll \frac{1}{\Delta \nu}, \quad (11)$$

then both the space-time and the space-frequency treatments will produce identical results. In Eq. (11) \mathbf{p}_1 and \mathbf{p}_2 are the radial vectors in the double-slit (DS) plane (see Fig. 8 in [13]). Accordingly, $|\mathbf{p}_2 - \mathbf{p}_1|$ corresponds to the double slit size. Using Eqs. (9) and (10) with Eq. (5), we have

$$\frac{J_{ij}(\mathbf{r}_1, \mathbf{r}_2, 0)}{\sqrt{I_i(\mathbf{r}_1)}\sqrt{I_j(\mathbf{r}_2)}} = \frac{W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega)}{\sqrt{S_i(\mathbf{r}_1, \omega)}\sqrt{S_j(\mathbf{r}_2, \omega)}}, \text{ for } (i, j) = (x, y); \quad (12)$$

i.e., the elements of the BCP matrix, normalized to intensities at the two points are identical to the elements of the CSD matrix, normalized to the spectral densities.

We investigate experimentally Eq. (12) for different polarizations of light. For xx diagonal element, thin rectangular sheet polarizers P_1 and P_2 (see Fig. 1), cut in x directions were put separately before slits at DS passing the x components of the optical field only (similar to [35]). This produced the interference pattern for x polarized light. Quantities $|\gamma_{xx}(\mathbf{r}_1, \mathbf{r}_2, 0)|$ and $|\mu_{xx}(\mathbf{r}_1, \mathbf{r}_2, \omega)|$ were measured for the central fringe with the help of Eq. (7) and their values were found nearly the same (≈ 0.21). For diagonal yy element, P_1 and P_2 were rotated by right angle (90°) producing the interference fringes for y polarized light. The magnitudes of the degrees of coherence $|\gamma_{yy}(\mathbf{r}_1, \mathbf{r}_2, 0)|$ and $|\mu_{yy}(\mathbf{r}_1, \mathbf{r}_2, \omega)|$ were also determined in the similar manner. These quantities were having nearly the same visibility (≈ 0.21) within experimental uncertainty. This showed that the normalized diagonal elements of both the BCP and the CSD matrices were having identical values within the quasi-monochromatic limit imposed by Eq. (11).

To determine the off-diagonal elements of these matrices, visibility measurement were performed by rotating one of the sheet polarizers (say P_2) by 90° (see Fig. 1). It allowed x and y electric field components of the beam to pass through different slits. As orthogonally polarized light beams do not interfere (called Fresnel and Arago laws [35]), we introduced a thin rectangular half wave plate H having optic axis at an angle 45° with the incident polarization of the beam, just after one of the slits in DS (see Fig. 1). This rotated the incident polarization of the beam by 90° , making both the beams (after the slits) with same planes of polarizations, enabling them to interfere. However, no interference fringes were produced for this (xy) polarization. The reason was that approximately no correlation was present in between the orthogonal electric field components of the unpolarized light beam. So the visibility of the interference fringes in space-time ($|\gamma_{xy}(\mathbf{r}_1, \mathbf{r}_2, 0)|$) and in space-frequency domain ($|\mu_{xy}(\mathbf{r}_1, \mathbf{r}_2, \omega)|$) measured using SP and PD (see Fig. 1) were found approximately zero. By interchanging the sheet polarizers and repeating the experiment, same results (no fringes) were obtained for yx polarization of the light. Quantities $|\gamma_{yx}(\mathbf{r}_1, \mathbf{r}_2, 0)|$ and $|\mu_{yx}(\mathbf{r}_1, \mathbf{r}_2, \omega)|$ were again measured having nearly 0% visibility. This infers the identical nature of the normalized off-diagonal elements of both the BCP and the CSD matrices, within the quasi-monochromatic limit [Eq. (11)]. These results obtained for the diagonal and the off-diagonal elements justify our claim which has been made about the equivalence of the elements of the BCP and CSD matrices, for a filtered thermal (unpolarized) beam of light [see Eqs. (11) and (12)].

It is worthwhile to mention here that in this experiment the double-slit (DS) size $|\mathbf{p}_2 - \mathbf{p}_1|$ was $400 \mu\text{m}$, which required FWHM $< 1.3 \text{ nm}$, satisfying Eq. (11). The above mentioned measurements were performed for light bandwidth around 0.9 nm . In strict terms,

for lower frequency spread of the source, the matching in the space-time and the space-frequency visibilities are better, and the inverse. Though we have measured the absolute values of the degrees of coherence only, yet the phase information can also be acquired using the methods well known in literature [18, 19]. So as long as Eq. (11) holds, either of the two treatments (space-time or space-frequency) could be used to characterize the polarization and the coherence properties simultaneously, at a pair of points in the cross-section of a random electromagnetic beam.

6. Conclusion

In summary, the relationship between the space-time and the space-frequency degrees of coherence is studied experimentally using a variable bandwidth source in Young's double-slit interference experiment. We found that by decreasing the bandwidth of light, the absolute value of the complex degree of coherence for a pair of points increases but doesn't approach to unity. The maximum value of this quantity determined at the central fringe is limited by its spectral visibility that remains unchanged in the process of filtering of light. However, the visibility of the off central fringes remains less than that for the central fringe. Another important result of this paper is the equivalence condition for the normalized elements of the BCP and the CSD matrices. We have shown that for a quasi-monochromatic (or sufficiently narrowband) electromagnetic beam, the normalized elements of the BCP and the CSD matrices own identical values. These findings might be applicable for the free-space communication studies and could pave the way for further research in the field of coherence and polarization.

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